JPE395 - Problem Set 1 Due Feb. 16, 11:59pm

Q1) Magnetic Isochrons

a) In both plate tectonic profiles, it appears as if the profile indicates the spreading (ridge) of oceanic crust. This is because new crust is being created, and the profiles are symmetric around a point (indicating the ridge location).

b) For profile 1, due to the symmetry around 250 km, the profile is compared with the one in the appendix and is estimated to be ~5 Ma on both ends.

For profile 2, the same process was applied to estimate ~3 Ma on the left side and ~4 Ma on the right.

c) For this part, only two points on each profile were selected. They are

P1: (5 Ma, 250 km) and (2 Ma, 100 km)

P2: (4 Ma, 200 km) and (3 Ma, 150 km).

These points were determined by the method used in part (b) of this problem.

Then, since $v = \frac{d}{t}$,

P1: $V_1 = \frac{250\text{ km}}{5\text{ Ma}} \simeq 50 \frac{\text{km}}{\text{Ma}} = 5\text{ cm/yr}$

$V_1' = \frac{100\text{ km}}{2\text{ Ma}} \simeq 50 \frac{\text{km}}{\text{Ma}} = 5\text{ cm/yr}$

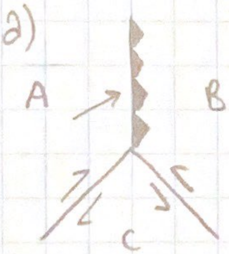
P2: $V_2 = \frac{150\text{ km}}{3\text{ Ma}} \simeq 50 \frac{\text{km}}{\text{Ma}} = 5\text{ cm/yr}$

$V_2' = \frac{200\text{ km}}{4\text{ Ma}} \simeq 50 \frac{\text{km}}{\text{Ma}} = 5\text{ cm/yr}$ (Half-Spreading Rates)

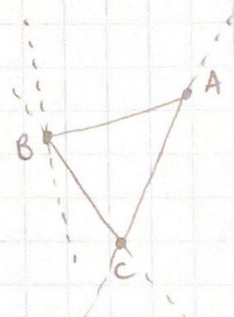
Thus the full spreading rate for both plate profiles appears to be ~ 10 cm/yr.

Q2) Triple Junction Stability

a)

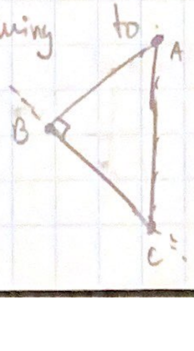


Corresponds to velocity diagram



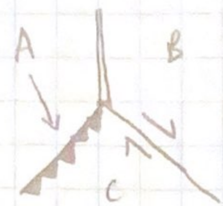
Where the velocity lines do not intersect.

Thus, this triple junction is unstable, however if plate A subducts under plate B at the same angle which A is transforming to C, this junction becomes stable:

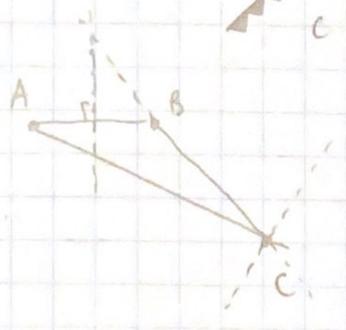


Since all three velocity lines intersect at 'c'.

b) Now, consider the RTF:



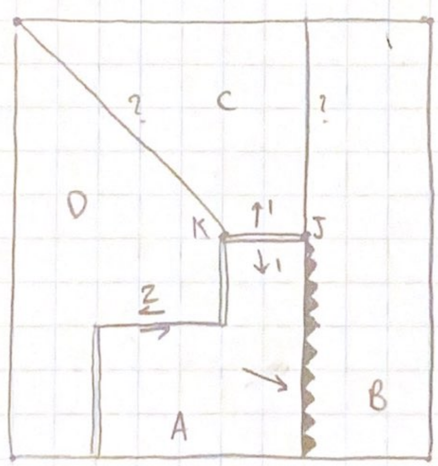
whose corresponding velocity diagram is



whose velocity lines never intersect.

There does not exist a geometry in which the velocity lines do intersect, and therefore this triple junction is always unstable.

Q3) Flat Earth Triple Junctions

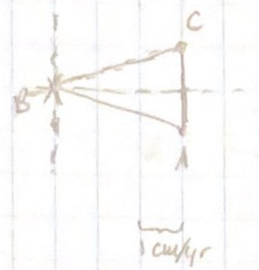


a) To determine ${}_{C}\vec{V}_{B}$, consider ${}_{A}\vec{V}_{C}$ and ${}_{A}\vec{V}_{B}$. Thus:

$${}_{C}\vec{V}_{B} = {}_{C}\vec{V}_{A} + {}_{A}\vec{V}_{B} = -2\hat{j} - 3\hat{i} + \hat{j}$$

$$= -3\hat{i} - \hat{j}.$$

For J to be a stable junction, consider the velocity diagram:



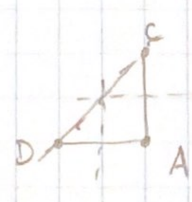
And thus for J to be stable, $\overline{BC}$ must be a trench, since C has a relative velocity towards B. Then, the trench must be parallel to the $\hat{i}$ direction.

b) ${}_D\vec{V}_C$ can be found via the same method.

Since ${}_A\vec{V}_D = -2\hat{i}$ and ${}_A\vec{V}_C = +2\hat{j}$,

then
$$ {}_D\vec{V}_C = {}_D\vec{V}_A + {}_A\vec{V}_C $$
$$ = 2\hat{i} + 2\hat{j}. $$

Then, the velocity diagram for K is



which is stable, because it must be that $\overline{CD}$ is a transform fault.

c) The velocity of K relative to B ${}_B\vec{V}_K$ is dependent on the half-spreading rates between A, D and C. Thus it is easy to determine that ${}_B\vec{V}_K = \hat{i} - \hat{j}$.

The magnitude of this vector is $\sqrt{2}$, so $|{}_B\vec{V}_K| = \sqrt{2}$ cm/yr

If the distance between K and B is 2000 km $\hat{i}$, 2000 km $\hat{j}$,

then since $v = \frac{d}{t} \rightarrow t = \frac{d}{v}$,

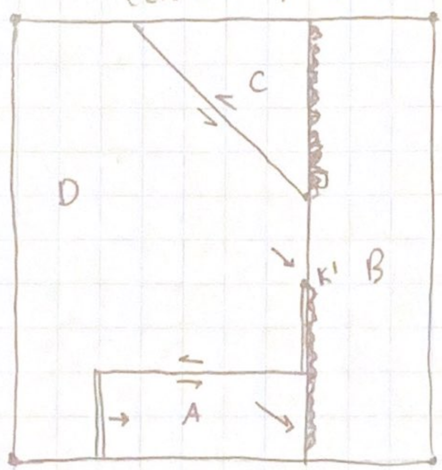
$$t = \frac{2'000 \text{ km}}{1 \text{ cm/yr}} \quad (\text{only in } +\hat{i} \text{ direction})$$

$$= \frac{2.0 \times 10^8 \text{ cm}}{1 \text{cm}} \text{ yr}$$

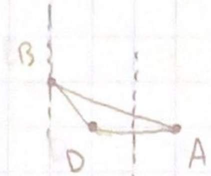
$$= 2.0 \times 10^8 \text{ yr}.$$

d) Once K hits plate B, we have

(2.0 × 10⁸ yr later)



What is the boundary DB so that K' is still stable?



Here, K' will never be stable. If K' were to be stable, either A/D or A/B boundaries should no longer be spreading or subducting, respectively.

For instance, if $\vec{V}_{AD} \to 0$, then

ie, A and D stop spreading and keep subducting under B.

Q4) Spherical Plate Motion

a) First, consider the expression for arc length given the radius of the earth R_e.

$$d = R_e \theta .$$

Now, for velocities rotating around a central axis (such as an Euler pole),

$$\vec{V} = \vec{\omega} \times \vec{r} \implies |V| = |\omega||R_e| \sin\theta .$$

Concatenating, we obtain that

$$|V| = \omega R_e \sin\left(\frac{d}{R_e}\right).$$

b) To determine ${}_{ant}\vec{\omega}_{aus}$, we can use NUVEL-1 to determine ${}_{aus}\vec{\omega}_{pac}$, ${}_{ant}\vec{\omega}_{pac}$, thus

$${}_{ant}\vec{\omega}_{aus} = {}_{ant}\vec{\omega}_{pc} + {}_{pc}\vec{\omega}_{aus}$$

$$= (0.000689 - 0.009344)\hat{x} + (-0.006541 - 0.000284)\hat{y}$$

$$+ (0.013676 - 0.016252)\hat{z} \quad \text{rad/Myr}$$

$= -0.00866\,\hat{x} \quad - 0.006825\,\hat{y} \quad - 0.002576\,\hat{z}$.

c) This part of the problem was completed using python. The values of $|\vec{v}|$ (spreading rate) are given in the NUVEL-1 table, however the distances d must be calculated using the haversine formula because we need arclength distance on a sphere:

$$a = \sin^2\left(\frac{\varphi_2 - \varphi_1}{2}\right) + \cos\varphi_1 \cos\varphi_2 \sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)$$

$$d = 2R_e \arctan\left(\frac{\sqrt{a}}{\sqrt{1-a}}\right).$$

where $\varphi_2 - \varphi_1$ is the change in latitude, in radians, and $\lambda_2 - \lambda_1$ is the change in longitude.

Defining this formula in python and letting

$R_e = 6371$ km

$\varphi_2 =$ spreading distance [i] latitude

$\varphi_1 = 13.2° \times \frac{\pi}{180°}$ (into radians) (Euler-Pole latitude)

$\lambda_2 =$ spreading distance [i] longitude

$\lambda_1 = 38.2° \times \frac{\pi}{180°}$ (into radians) (Euler-Pole Longitude)

the following table is extracted for the spreading velocity $|v|$ and great circle distance d:

(mm/yr) Spreading Velocity |v|	(km) Great Circle Distance to Euler Pole d
56	5552.87
57	5673.47
58	5716.41
59	5748.07
63	5873.88
60	5987.05
61	6000.93
60	6187.69
63	6376.52
63	6527.57
65	6828.82
67	6934.85
69	7321.51
70	7501.13
73	8128.87
72	8135.44
74	8371.95
73	8482.85
71	8829.80
73	9879.46
74	10000.28
75	10143.97
76	10458.33
75	10680.66
76	10902.38
75	11150.59
73	11331.14
73	11385.28
75	11392.36
73	11406.52
73	11435.89
73	11534.99
73	11613.60
72	11965.56
70	12278.64
68	12801.36
69	12819.74
68	12847.82

d) Once plotted, it was concluded that the great circle distance and spreading velocity magnitude data plotted did match the expected trend of $|v| = \omega R_e \sin\left(\frac{d}{R_e}\right)$ as predicted in (a). Although most of the data points did not lie right on the curve, the still followed the appropriate trend, and thus this was sufficient.

Q5) Spherical Earth Plate Velocities

(a) First, to determine the relative velocity between the two plates, we should consider their relative angular velocities:

$$_{af}\vec{\omega}_{pc} = 0.002401\,\hat{x} - 0.007939\,\hat{y} + 0.013892\,\hat{z}$$

$$_{sa}\vec{\omega}_{pc} = 0.000472\,\hat{x} - 0.006355\,\hat{y} + 0.009100\,\hat{z}.$$

Then

$$_{SA}\vec{\omega}_{AF} = {}_{SA}\vec{\omega}_{PC} + {}_{PC}\vec{\omega}_{AF}$$

$$= -0.001929\,\hat{x} + 0.001584\,\hat{y} - 0.004792\,\hat{z}.$$

To determine $_{SA}\vec{V}_{AF} = {}_{SA}\vec{\omega}_{AF} \times \vec{r}$ in cartesian (global) coordinates, then, we determine $\vec{r}$:

$$\vec{r} = (R_e \sin\theta \cos\varphi,\ R_e \sin\theta \sin\varphi,\ R_e \cos\theta)$$

where

R_e is the earth radius (6371 km),

θ is colatitude $(\frac{\pi}{2} - 0)$, and

φ is azimuthal $\left(-\frac{\pi}{3}\right)$.

Thus $\vec{r} = (3185, -5517.45, 0)$.

Then ${}_{SA}\vec{V}_{AF} = \det \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ -0.001429 & 0.001584 & -0.004792 \\ 3185 & -5517.45 & 0 \end{pmatrix}$

$= (-26.43961, -15.264916, 5.59732489)$

by Python, in global coordinates.

To find $\vec{V}$ in local polar coordinates, we convert back by applying the coordinate transformation

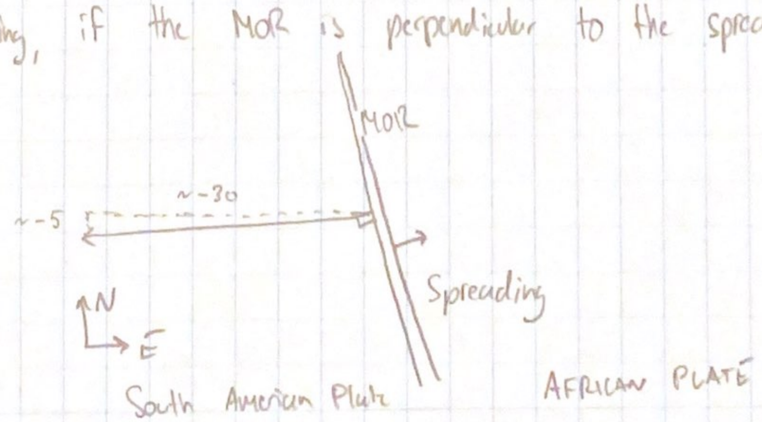
$$\begin{bmatrix} V_r \\ V_\theta \\ V_\varphi \end{bmatrix} = \begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{pmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}.$$

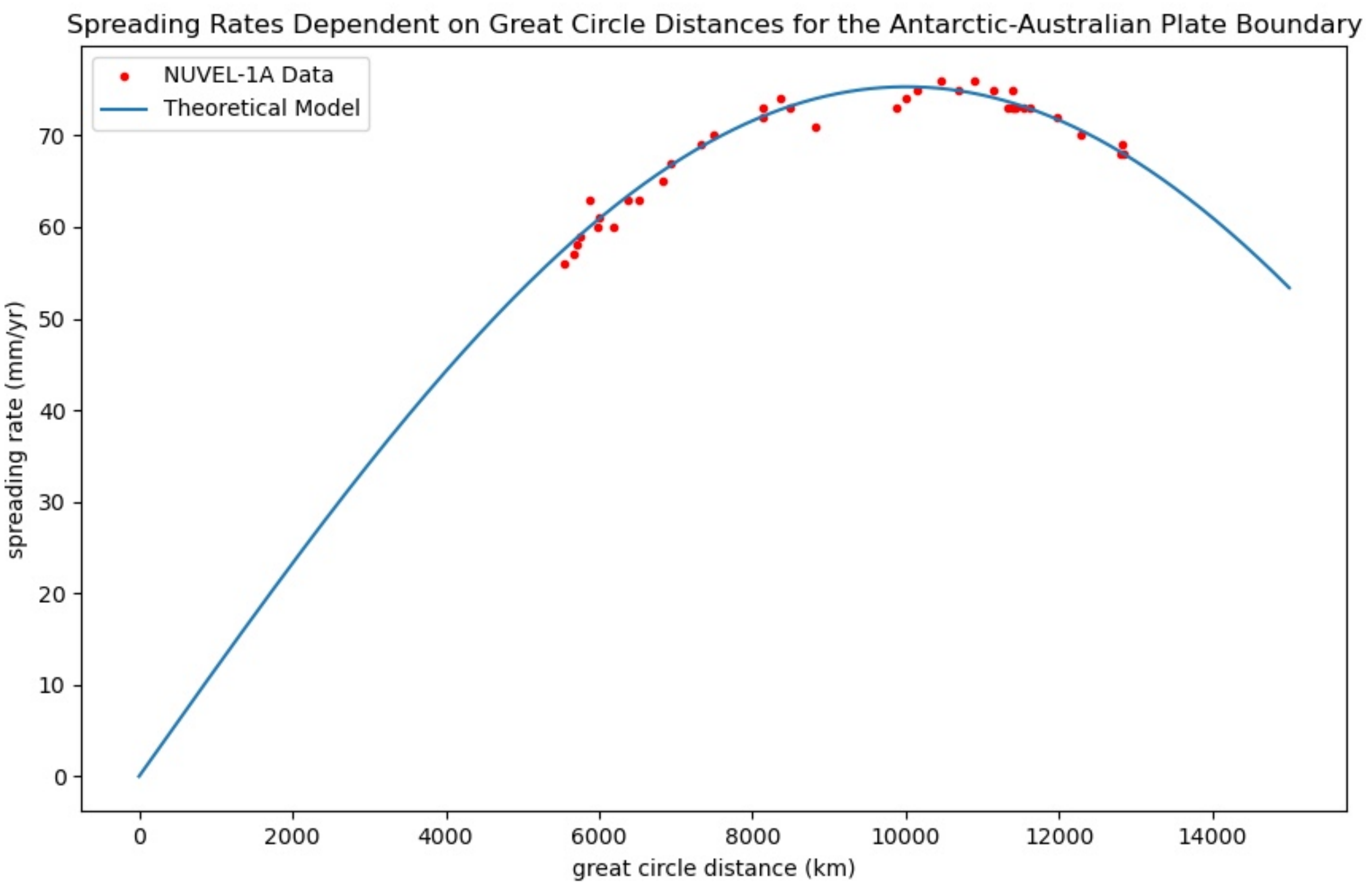
With $\theta = \frac{\pi}{2}$, $\varphi = -\frac{\pi}{3}$, we have

$$\begin{pmatrix} V_r \\ V_\theta \\ V_\varphi \end{pmatrix} = \begin{pmatrix} 0.5 & -\sqrt{3}/2 & 0 \\ 0 & 0 & -1 \\ \sqrt{3}/2 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} -26.43961 \\ -15.264916 \\ 5.59732489 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -5.597 \\ -30.529 \end{pmatrix} \text{ in mm/yr.}$$

b) Then, drawing, if the MOR is perpendicular to the spreading direction,




Spreading Rates Dependent on Great Circle Distances for the Antarctic-Australian Plate Boundary
NUVEL-1A Data
Theoretical Model
spreading rate (mm/yr)
0
10
20
30
40
50
60
70
great circle distance (km)
0
2000
4000
6000
8000
10000
12000
14000

ps1code.py ● spreading rate data.txt

ps1code.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt


data = np.loadtxt('spreading rate data.txt', skiprows=1, unpack=True)


latitude = data[0] * (np.pi/180)
longitude = data[1] * (np.pi/180)
spreadrate = data[2] #km/Ma


    #for the australian-antarctic plate pair:

omega = np.sqrt((0.000689-0.009349)**2 + (-0.006541-0.000284)**2 + (0.013676 - 0.016252)**2)+0.0005
lat = 13.2 * (np.pi/180)
long = 38.2 * (np.pi/180)
R_earth = 6371 #km
xx = np.linspace(0, 15000, 15000)

def haversine_distance(lat1, lat2, long1, long2, radius):
    d = ((np.sin(np.abs(lat2-lat1)/2))**2 + np.cos(lat1)*np.cos(lat2)*(np.sin(np.abs(long2-long1)/2))**2  )
    c = 2*radius*np.arctan(np.sqrt(d)/(np.sqrt(1-d)))
    return c

def spreading_rate(omega, radius, d):
    v = omega*radius*np.sin(d/radius)
    return np.abs(v)


eulerpole_distances = np.zeros(len(latitude))
for i in range(len(latitude)):
    dist = haversine_distance(lat, latitude[i], long, longitude[i], R_earth)
    eulerpole_distances[i] = dist



plotting=True
if plotting:
    plt.title('Spreading Rates Dependent on Great Circle Distances for the Antarctic-Australian Plate Boundary')
    plt.plot(eulerpole_distances,spreadrate, 'r.', label='NUVEL-1A Data')
```

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ps1code.py > ...

```python

plotting=True
if plotting:
    plt.title('Spreading Rates Dependent on Great Circle Distances for the Antarctic-Australian Plate Boundary')
    plt.plot(eulerpole_distances,spreadrate, 'r.', label='NUVEL-1A Data')
    plt.plot(xx, spreading_rate(omega, R_earth, xx), label='Theoretical Model' )
    plt.xlabel('great circle distance (km)')
    plt.ylabel('spreading rate (mm/yr)')
    plt.legend(loc='best')
    plt.show()
plt.close()


table = False
if table:
    print('the great circle distance d and spreading rate |v| table is:')
    for i in range(len(eulerpole_distances)):
        print(np.round(eulerpole_distances[i],2),'    ',  spreadrate[i])



def coordinate_conversion(R, phi, theta):
    rx = R*np.cos(phi)*np.sin(theta)
    ry = R*np.sin(phi)*np.sin(theta)
    rz = R*np.cos(theta)
    return (rx, ry, rz)

omega_saaf = (0.000472 - 0.002401, -0.006355+0.007939, 0.009100-0.013892)
r = coordinate_conversion(R_earth, -np.pi/3, np.pi/2)

q=-r[1]*omega_saaf[2], r[0]*omega_saaf[2], (r[1]*omega_saaf[0] - r[0]*omega_saaf[1])

print(omega_saaf)


print(r)

print(0.5*q[0] - np.sqrt(3)*q[1]/2, -q[2],0.5*q[1] + np.sqrt(3)*q[0]/2 )


```

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spreading rate data.txt

```
Lat long    spreading rate
-25.81  70.23   56
-26.17  71.57   57
-26.37  71.96   58
-26.67  72.07   59
-27.70  72.70   63
-28.00  74.00   60
-28.00  74.20   61
-29.50  75.20   60
-31.30  75.90   63
-32.20  77.10   63
-34.80  78.60   65
-36.00  78.80   67
-40.90  78.80   69
-41.30  81.30   70
-42.40  90.00   73
-42.40  90.10   72
-43.50  92.60   74
-44.00  93.80   73
-46.90  96.40   71
-49.75  110.20  73
-50.10  111.80  74
-50.00  114.00  75
-49.80  118.70  76
-49.80  121.90  75
-50.00  125.00  76
-50.10  128.50  75
-50.40  131.00  73
-50.20  131.80  73
-50.20  131.90  75
-50.20  132.10  73
-50.30  132.50  73
-50.30  133.90  73
-50.40  135.00  73
-52.00  140.00  72
-54.70  145.00  70
-62.50  157.80  68
-62.40  158.10  69
-62.30  158.60  68
```

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