PHY256 PS4

6.1) Constants (found using Google)

$m_e = 9.11 \times 10^{-31}$ kg $\quad m_p = 1.67 \times 10^{-27}$ kg

$h_{eV} = 4.135\,663 \times 10^{-15}$ eV·s

$h_J = 6.6261 \times 10^{-34}$ J·s $\qquad \lambda_{dB} = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2K_E m}}$

$m_{buckyball} = 1.1952 \times 10^{-23}$ kg

$m_{oxygen} = 5.31 \times 10^{-20}$ kg $\qquad E_{oxygen\ @\ 20°C} = 6.21 \times 10^{-21}$ J

$m_{raindrop} = 3.4 \times 10^{-5}$ kg $\qquad V_{raindrop} \simeq 10$ m/s

$m_{me} \simeq 75$ kg , I walk about 2.6 m/s.

a) $$\lambda_{dB} = \frac{4.135663 \times 10^{-15} \text{ eV·s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(3\text{eV})}} = \boxed{1.7689 \text{ m}}$$

b) $$\lambda_{dB} = \frac{4.135663 \times 10^{-15} \text{ eV·s}}{\sqrt{2(1.67 \times 10^{-27} \text{ eV})(7 \times 10^{6} \text{ eV})}} = \boxed{2.7047 \times 10^{-5} \text{ m}}$$

c) $$\lambda_{dB} = \frac{6.6261 \times 10^{-34} \text{ J·s}}{(1.1952 \times 10^{-23} \text{ kg})(200 \frac{m}{s})} = \boxed{2.7719 \times 10^{-13} \text{ m}}$$

d) $$\lambda_{dB} = \frac{6.6261 \times 10^{-34} \text{ J·s}}{\sqrt{2(5.31 \times 10^{-20} \text{ kg})(6.21 \times 10^{-21} \text{ J})}} = \boxed{2.5802 \times 10^{-14} \text{ m}}$$

e) $$\lambda_{dB} = \frac{6.6261 \times 10^{-34} \text{ J·s}}{(3.4 \times 10^{-5} \text{ kg})(10 \frac{m}{s})} = \boxed{1.9489 \times 10^{-30} \text{ m}}$$

f) $$\lambda_{dB} = \frac{6.6261 \times 10^{-34} \text{ J·s}}{(74.85 \text{ kg})(2.6 \frac{m}{s})} = \boxed{3.4048 \times 10^{-36} \text{ m}}$$

(a), (b), (c) will play a role in QM because their de Broglie wavelengths are longer ⇒ less momentum ⇒ less classical.
(d), (e), (f) will behave classically because their dB. wavelengths are shorter ⇒ more momentum . . .

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6.2.ii) $\psi(x) = A x e^{-x^2/a^2}$.

In this question, I will abuse notation and let '$\int_{-\infty}^{\infty}$' $:= \int_{\mathbb{R}}$.

a) We require $1 = \int_{\mathbb{R}} dx\, |\psi(x)|^2 = A^2 \int_{\mathbb{R}} dx\, x^2 e^{-2x^2/a^2}$.

Now $1 = A^2 \int_{\mathbb{R}} dx\, x \cdot x\, e^{-2x^2/a^2}$ $\quad u = -\frac{2x^2}{a^2}$, $du = -\frac{4x}{a^2}dx$

$\Rightarrow x\,dx = -\frac{a^2}{4}du$.

$$= A^2\left[\cancelto{0}{x e^{-\frac{2x^2}{a^2}}\left(-\frac{a^2}{4}\right)\Bigg|_{\mathbb{R}}} + \frac{a^2}{4}\int_{\mathbb{R}} dx\, e^{-\frac{2x^2}{a^2}}\right]$$

$$= A^2\left[\frac{a^2}{4}\int_{\mathbb{R}} dx\, e^{-\frac{2x^2}{a^2}}\right]$$

Know $\int dx\, e^{-p^2x^2} = \frac{\sqrt{\pi}}{p}$.

Let $p = \frac{\sqrt{2}}{a}$.

$$= \frac{A^2 a^2}{4}\left[a\sqrt{\frac{\pi}{2}}\right] = A^2 \frac{a^3}{4}\sqrt{\frac{\pi}{2}}$$

$$\Longrightarrow A = \frac{2}{a^{3/2}}\left(\frac{2}{\pi}\right)^{1/4}.$$

Thus $\boxed{\psi(x) = \frac{2}{a^{3/2}}\left(\frac{2}{\pi}\right)^{1/4} x e^{-x^2/a^2}}$

b) $\langle \hat{x} \rangle = \int_{\mathbb{R}} dx\, \psi^*(x)\, x\, \psi(x) = \int_{\mathbb{R}} dx\, x \cdot \left[\frac{2}{a^{3/2}}\left(\frac{2}{\pi}\right)^{1/4} x e^{-x^2/a^2}\right]^2$

$$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx\, x^3 e^{-2x^2/a^2}$$

$= 0$ $\quad$ By properties of odd functions, since $\int_{\mathbb{R}} dx\, x^3 e^{-2x^2/a^2} = 0$.

Thus $\boxed{\langle \hat{x} \rangle = 0.}$

c) To find Δx, we must find $\langle \hat{x}^2 \rangle$. From (b), $\langle \hat{x} \rangle = 0$.

$$\langle \hat{x}^2 \rangle = \int_{\mathbb{R}} dx\ \psi^*(x)\ x^2\ \psi(x) = \int_{\mathbb{R}} dx\ x^2 \left[\frac{2}{a^{3/2}} \left(\frac{2}{\pi}\right)^{1/4} x e^{-x^2/a^2} \right]^2$$

$$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx\ x^4 e^{-2x^2/a^2}$$

By parts: $\int u\,dv = \det \begin{pmatrix} u & du \\ \int v & v \end{pmatrix}$

[1] $u = x^3 \quad du = 3x^2\,dx$

$dv = x e^{-\frac{2x^2}{a^2}} dx \quad v = -\frac{a^2}{4} e^{-\frac{2x^2}{a^2}}$

$$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \left[\cancel{-\frac{a^2}{4} x^3 e^{-\frac{2x^2}{a^2}} \Big|_{\mathbb{R}}} \to 0 + \frac{3a^2}{4} \int_{\mathbb{R}} dx\ x^2 e^{-\frac{2x^2}{a^2}} \right]$$

[2] $u = x \quad du = dx$

$dv = x e^{-\frac{2x^2}{a^2}} dx \quad v = -\frac{a^2}{4} e^{-\frac{2x^2}{a^2}}$

$$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \left[0 + \frac{3a^2}{4} \left[\cancel{-\frac{a^2}{4} x e^{-\frac{2x^2}{a^2}} \Big|_{\mathbb{R}}} \to 0 + \frac{a^2}{4} \int_{\mathbb{R}} dx\ e^{-\frac{2x^2}{a^2}} \right] \right]$$

$$= \frac{4}{\cancel{a^3}} \cancel{\sqrt{\frac{2}{\pi}}} \left[\frac{3 \cancel{a^4}^{2}}{16} \left[\cancel{a} \cancel{\sqrt{\frac{\pi}{2}}} \right] \right]$$

Thus $\langle \hat{x}^2 \rangle = \frac{3}{4} a^2. \implies \Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$

So $\boxed{\Delta x = \frac{\sqrt{3}}{2} a.}$

d) $\psi(x) = \frac{2}{a^{3/2}} \left(\frac{2}{\pi}\right)^{1/4} x e^{-x^2/a^2}$. Find $\mathbb{P}_{0<x<a}(x)$.

We have $\mathbb{P}_{0<x<a} = \int_0^a dx\, |\psi(x)|^2 = \int_0^a dx \left(\frac{4}{a^3}\sqrt{\frac{2}{\pi}}\, x^2 e^{-2x^2/a^2} \right)$

$$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \int_0^a dx\, x^2 e^{-2x^2/a^2}$$

[By Parts] $$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \left[-\frac{a^2}{4} x e^{-\frac{2x^2}{a^2}} \Big|_0^a + \frac{a^2}{4} \int_0^a dx\, e^{-\frac{x^2}{a^2}} \right]$$

Let $u = \frac{x}{a}$. $\to u(a) = 1$.

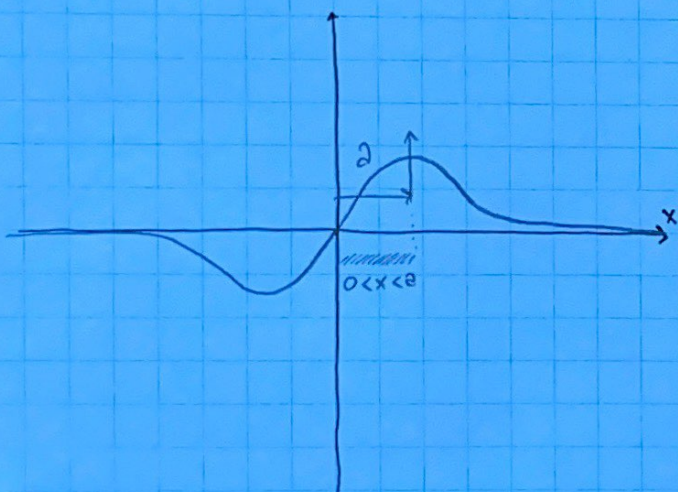
$du = \frac{1}{a}\, dx$. $$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \left[-\frac{a^2}{4} x e^{-\frac{2x^2}{a^2}} \Big|_0^a + \frac{a^2}{4} \int_0^1 du \cdot a\, e^{-u^2} \right]$$

$$= \frac{\cancel{4}}{\cancel{a^3}}\sqrt{\frac{2}{\pi}} \left[-\frac{\cancel{a^2}}{\cancel{4}} \left[\cancel{a} e^{-2} - \cancel{a}\right] + \frac{\cancel{a^3}}{\cancel{4}} \operatorname{erf}(1) \cdot \frac{\sqrt{\pi}}{2} \right]$$

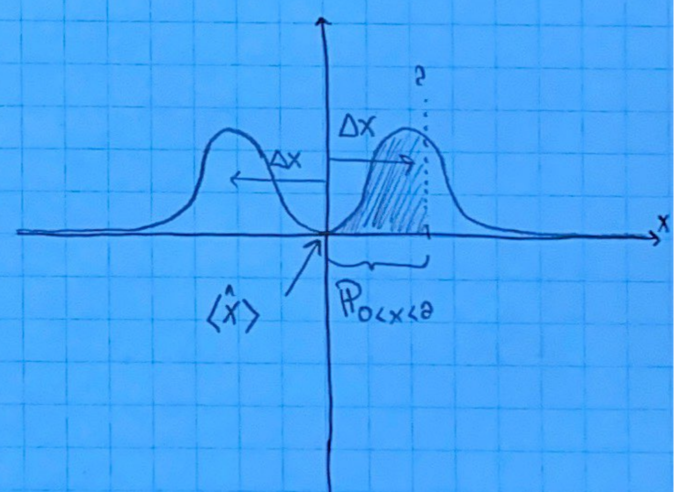
$$= \sqrt{\frac{2}{\pi}} \left(-e^{-2} + 1 + \frac{\sqrt{\pi}}{2} \operatorname{erf}(1) \right)$$

$$\boxed{\mathbb{P}_{0<x<a} = \frac{\sqrt{2}}{2} \operatorname{erf}(1) + \sqrt{\frac{2}{\pi}} \left(1 - e^{-2}\right)}$$

e) I: $\psi(x)$



II: $|\psi(x)|^2$



f) $\hat{P} := -i\hbar \frac{\partial}{\partial x}$. $\rightsquigarrow$ $\langle \psi | \hat{p} | \psi \rangle \to \int_{\mathbb{R}} dx\, \psi^*(x) \left[\frac{\partial}{\partial x} \psi(x) \right]$.

$$\langle \hat{P} \rangle = \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx \, \left(xe^{-x^2/a^2}\right) \left[\frac{\partial}{\partial x} xe^{-x^2/a^2}\right] = \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx \, \left(xe^{-x^2/a^2}\right)\left(e^{-x^2/a^2} - \frac{2x^2}{a^2} e^{-2x^2/a^2}\right)$$

$$= \frac{4}{a^3}\sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx \; xe^{-2x^2/a^2} - \frac{2}{a^2} x^3 e^{-2x^2/a^2}$$

$\boxed{\langle \hat{P} \rangle = 0}$ By properties of odd functions, since

$$\int_{\mathbb{R}} dx \; xe^{-2x^2/a^2} = \int_{\mathbb{R}} dx \; x^3 e^{-2x^2/a^2} = 0.$$

g) To find ΔP, we must find $\langle \hat{p}^2 \rangle$. $\hat{p}^2 = \left(-i\hbar \frac{\partial}{\partial x}\right)^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$.

$$\langle \psi | \hat{p}^2 | \psi \rangle \rightsquigarrow -\hbar^2 \int_{\mathbb{R}} dx \left(\frac{2}{a^{3/2}} \left(\frac{2}{\pi}\right)^{1/4} x e^{-x^2/a^2} \right) \left[\frac{\partial^2}{\partial x^2} \left(\frac{2}{a^{3/2}} \left(\frac{2}{\pi}\right)^{1/4} x e^{-x^2/a^2} \right) \right]$$

$$= -\hbar^2 \frac{4}{a^3} \sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx \left(x e^{-x^2/a^2} \right) \frac{\partial^2}{\partial x^2} \left[x e^{-x^2/a^2} \right]$$

Aside I: $\frac{\partial^2}{\partial x^2}\left[x e^{-x^2/a^2}\right] = \frac{\partial}{\partial x}\left[e^{-x^2/a^2} - \frac{2x^2}{a^2} e^{-x^2/a^2} \right]$

$$= -\frac{2x}{a^2} e^{-x^2/a^2} - \frac{4x}{a^2} e^{-x^2/a^2} + \frac{4x^3}{a^4} e^{-x^2/a^2}$$

$$= \left(\frac{4x^3}{a^4} - \frac{6x}{a^2} \right) e^{-x^2/a^2}.$$

$$\Longrightarrow -\hbar^2 \frac{4}{a^3} \sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx \left(x e^{-x^2/a^2} \right) \left(\frac{4x^3}{a^4} - \frac{6x}{a^2} \right) e^{-x^2/a^2}$$

$$= -\hbar^2 \frac{4}{a^3} \sqrt{\frac{2}{\pi}} \int_{\mathbb{R}} dx\, e^{-2x^2/a^2} \left(\frac{4x^4}{a^4} - \frac{6x^2}{a^2} \right)$$

$$= -\hbar^2 \frac{4}{a^3} \sqrt{\frac{2}{\pi}} \left\{ \frac{4}{a^4} \int_{\mathbb{R}} dx\, x^4 e^{-2x^2/a^2} - \frac{6}{a^2} \int_{\mathbb{R}} dx\, x^2 e^{-2x^2/a^2} \right\}$$

Aside II:

$I_1 = \int_{\mathbb{R}} dx\, x^2 e^{-2x^2/a^2}$ $\quad u = x \quad du = dx$ $\quad dv = x e^{-2x^2/a^2} dx \quad v = -\frac{a^2}{4} e^{-2x^2/a^2}$ [By Parts]

$$= \cancel{-\frac{a^2}{4} x e^{-2x^2/a^2}}\Big|_{\mathbb{R}}^{\to 0} + \frac{a^2}{4} \int_{\mathbb{R}} dx\, e^{-2x^2/a^2} = \frac{a^2}{4}\left(a\sqrt{\frac{\pi}{2}} \right) = \frac{a^3}{4}\sqrt{\frac{\pi}{2}}.$$

$I_2 = \int_{\mathbb{R}} dx\, x^4 e^{-2x^2/a^2}$ $\quad u = x^3 \quad du = 3x^2 dx$ $\quad dv = x e^{-2x^2/a^2} dx \quad v = -\frac{a^2}{4} e^{-2x^2/a^2}$ [By Parts]

$$= \cancel{-\frac{a^2}{4} x^3 e^{-2x^2/a^2}}\Big|_{\mathbb{R}}^{\to 0} + \frac{3a^2}{4} \int_{\mathbb{R}} dx\, x^2 e^{-2x^2/a^2} = \frac{3a^2}{4} I_1.$$

$$= \frac{3a^2}{4}\left(\frac{a^3}{4}\sqrt{\frac{\pi}{2}} \right) = \frac{3a^5}{16}\sqrt{\frac{\pi}{2}}$$

Now

$$= -\hbar^2 \frac{4}{a^3}\sqrt{\frac{2}{\pi}}\left\{\frac{4}{a^4}\left(\frac{3a^5}{16}\sqrt{\frac{\pi}{2}}\right) - \frac{6}{a^2}\left(\frac{a^3}{4}\sqrt{\frac{\pi}{2}}\right)\right\}$$

$$= -\hbar^2 \frac{4}{a^3}\sqrt{\frac{2}{\pi}}\left\{\frac{3}{4}a\sqrt{\frac{\pi}{2}} - \frac{6}{4}a\sqrt{\frac{\pi}{2}}\right\} = \cancel{-}\hbar^2 \frac{\cancel{4}}{a^3}\cancel{\sqrt{\frac{2}{\pi}}}\left\{\cancel{-}\frac{3}{\cancel{4}}a\cancel{\sqrt{\frac{\pi}{2}}}\right\}$$

$$= \hbar^2 \frac{3}{a^2}.$$

This implies that $\Delta P = \sqrt{\langle \hat{P}^2 \rangle - \langle \hat{P} \rangle^2}$

So $\boxed{\Delta P = \sqrt{3}\,\frac{\hbar}{a}.}$

h) Uncertainty Principle: $\Delta x \, \Delta P \geq \frac{\hbar}{2}$

From (c), $\Delta x = \frac{\sqrt{3}}{2} a$

From (g), $\Delta P = \sqrt{3}\,\frac{\hbar}{a}$.

Thus $\left(\frac{\sqrt{3}}{2}a\right)\left(\sqrt{3}\,\frac{\hbar}{a}\right) = \boxed{\frac{3}{2}\hbar} > \frac{\hbar}{2}$.

<u>Therefore yes, this satisfies the uncertainty principle.</u>

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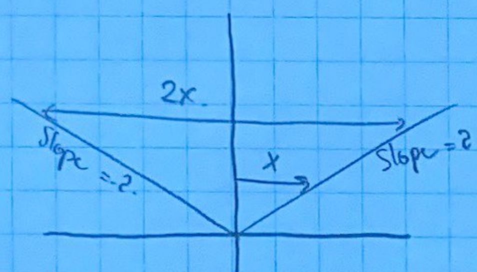
6.17) $V(x) = a|x|$.

We have $\Delta p \Delta x \geq \frac{\hbar}{2}$, so $p_{min} \simeq \frac{\hbar}{2\Delta x}$.

This implies that $E_{min} = \frac{p_{min}^2}{2m} = \frac{\hbar^2}{4\Delta x^2 m}$.



$|x| = \frac{V(x)}{a}$.

Will have some minimum E (E_{min}) at some Δx_{min}.

Thus $\Delta x = 2 \cdot \frac{V(x)}{a}$, which is the width at some V.

$\Longrightarrow \Delta x_{min} = \frac{2}{a} E_{min}$.

So $E_{min} = \frac{\hbar^2}{4\left(\frac{2}{a}E_{min}\right)^2 m} = \frac{\hbar^2 a^2}{8 E_{min}^2 m}$

$\Longrightarrow E_{min}^3 = \frac{\hbar^2 a^2}{8m} \Longrightarrow \boxed{E_{min} = \frac{1}{2}\left(\frac{\hbar^2 a^2}{m}\right)^{1/3}}$.

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5.11) $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$ [G.S. of $x=L$]

$\varphi_1(x) = \sqrt{\frac{2}{3L}} \sin\left(\frac{\pi x}{3L}\right)$ [G.S of $x=3L$]

$\varphi_2(x) = \sqrt{\frac{2}{3L}} \sin\left(\frac{2\pi x}{3L}\right)$ [E.S. of $x=3L$].

$$\sqrt{P_{\psi\to\varphi_1}} = \int_0^L dx \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \sqrt{\frac{2}{3L}} \sin\left(\frac{\pi x}{3L}\right) = \frac{2}{\sqrt{3}L}\int_0^L dx \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi x}{3L}\right)$$

$$= \frac{2}{\sqrt{3}L}\int_0^L dx \left[\frac{1}{2}\left(\cos\left(\frac{3\pi x - \pi x}{3L}\right) - \cos\left(\frac{3\pi x + \pi x}{3L}\right)\right)\right]$$

$$= \frac{1}{\sqrt{3}L}\int_0^L dx \left[\cos\left(\frac{2\pi x}{3L}\right) - \cos\left(\frac{4\pi x}{3L}\right)\right]$$

$$= \frac{1}{\sqrt{3}L}\left[\frac{3L}{2\pi}\sin\left(\frac{2\pi x}{3L}\right)\Big|_0^L - \frac{3L}{4\pi}\sin\left(\frac{4\pi x}{3L}\right)\Big|_0^L\right]$$

$$= \frac{1}{\sqrt{3}L}\left[\frac{3L}{2\pi}\frac{\sqrt{3}}{2} + \frac{3L}{4\pi}\frac{\sqrt{3}}{2}\right] = \frac{3}{4\pi} + \frac{3}{8\pi} = \frac{9}{8\pi}.$$

Thus $\boxed{P_{\psi\to\varphi_1} = \frac{81}{64\pi^2}}$. To first ground state.

$$\sqrt{P_{\psi\to\varphi_2}} = \int_0^L dx \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \sqrt{\frac{2}{3L}} \sin\left(\frac{2\pi x}{3L}\right) = \frac{2}{\sqrt{3}L}\int_0^L dx \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{3L}\right)$$

$$= \frac{2}{\sqrt{3}L}\int_0^L dx \cdot \frac{1}{2}\left[\cos\left(\frac{3\pi x - 2\pi x}{3L}\right) - \cos\left(\frac{3\pi x + 2\pi x}{3L}\right)\right]$$

$$= \frac{1}{\sqrt{3}L}\int_0^L dx \cos\left(\frac{\pi x}{3L}\right) - \cos\left(\frac{5\pi x}{3L}\right)$$

$$= \frac{1}{\sqrt{3}L}\left[\frac{3L}{\pi}\sin\left(\frac{\pi x}{3L}\right)\Big|_0^L - \frac{3L}{5\pi}\sin\left(\frac{5\pi x}{3L}\right)\Big|_0^L\right] = \frac{1}{\sqrt{3}L}\left[\frac{3L}{\pi}\cdot\frac{\sqrt{3}}{2} + \frac{3L}{5\pi}\cdot\frac{\sqrt{3}}{2}\right]$$

$$= \frac{3}{2\pi} + \frac{3}{10\pi} = \frac{18}{10\pi}.$$

Thus $\boxed{P_{\psi\to\varphi_2} = \frac{324}{100\pi^2}}$.

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5.25) We have $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) - \beta\delta(x)\psi(x) = E\psi(x)$.

When $x \neq 0$, $E < 0$, thus $\psi''(x) = -\frac{2m}{\hbar^2}E\psi(x)$

So $\psi(x) = \begin{cases} Ae^{-kx} & x < 0 \\ Be^{kx} & x > 0 \end{cases}$

to satisfy the normalization condition, where $k = \frac{\sqrt{-2mE}}{\hbar}$. $(E<0)$ $(k \in \mathbb{R})$.

Since ψ must be continuous at $x=0$, $A=B$. Since $V(0) = -\infty$, it suffices to only consider ψ and not ψ'. [From Q: 5.24]

Thus $\psi(x) = \begin{cases} Ae^{-kx} & x \leq 0 \\ Ae^{kx} & x \geq 0 \end{cases}$.

As in Q: 5.24, we must find the bound states at $x=0$ by integrating Schrödinger's equation over a small interval $[-a, a]$ then taking $\lim_{a\to 0}$:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) - \beta\delta(x)\psi(x) = E\psi(x)$$

$$\rightsquigarrow -\frac{\hbar^2}{2m}\int_{-a}^{a}dx\,\frac{\partial^2}{\partial x^2}\psi(x) - \beta\int_{-a}^{a}dx\,\delta(x)\psi(x) = E\int_{-a}^{a}dx\,\psi(x).$$

$$\Rightarrow -\frac{\hbar^2}{2m}\left[\frac{\partial\psi}{\partial x}\bigg|_{a} - \frac{\partial\psi}{\partial x}\bigg|_{-a}\right] - \beta\psi(0) = E\int_{-a}^{a}dx\,\psi(x).$$

Now $$-\frac{\hbar^2}{2m}\lim_{a\to 0}\left[\frac{\partial\psi}{\partial x}\bigg|_{a} - \frac{\partial\psi}{\partial x}\bigg|_{-a}\right] - \beta\psi(0) = \cancelto{0}{E\lim_{a\to 0}\int_{-a}^{a}dx\,\psi(x)}$$

$$\Rightarrow -\frac{\hbar^2}{2m}\left[-\cancel{A}k - (\cancel{A}k)\right] = \beta\cancel{A}$$

Thus $k = \frac{m\beta}{\hbar^2}$.

Now, if $K = \frac{\sqrt{-2mE}}{\hbar} = \frac{m\beta}{\hbar^2}$,

then $\frac{-2\cancel{m}E}{\cancel{\hbar^2}} = \frac{m^{\cancel{2}}\beta^2}{\hbar^{\cancel{4}2}} \Rightarrow -2E = \frac{m\beta^2}{\hbar^2}$, so $E = -\frac{m\beta^2}{2\hbar^2}$.

So $\psi(x) = Ae^{-\frac{m\beta}{\hbar^2}|x|}$ since this contains the solution for when $x \geq 0$ and $x \leq 0$.

Normalize ψ:

$$1 = \int_{\mathbb{R}} dx\, |\psi(x)|^2 = A^2 \int_{\mathbb{R}} dx\, e^{-2K|x|} = 2 \cdot A^2 \int_0^\infty dx\, e^{-2Kx}$$

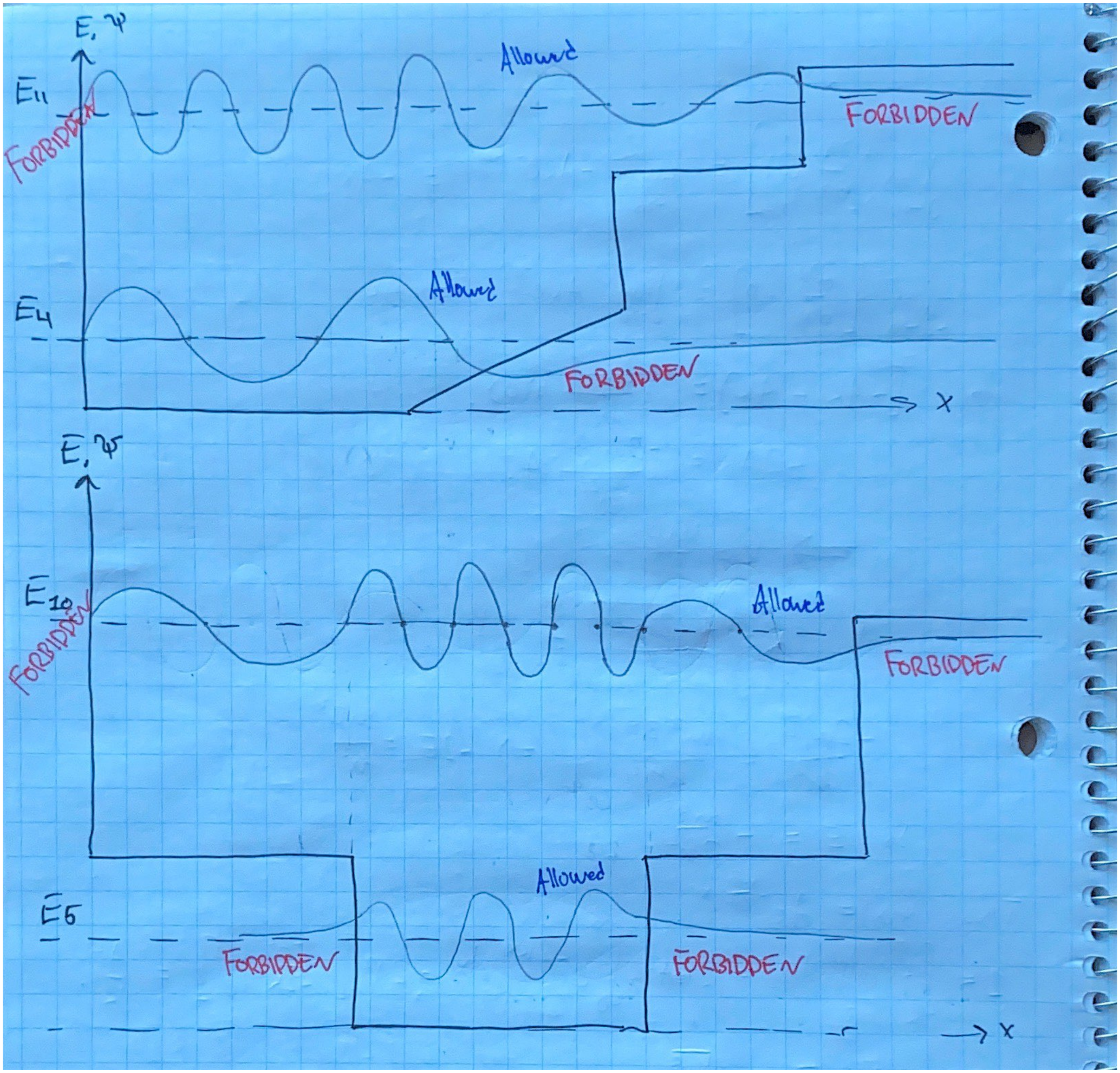
(ψ is symmetrical) ($x \geq 0$ here).

$$= 2A^2 \left[\frac{e^{-2Kx}}{-2K} \Big|_0^\infty \right] = 2A^2 \left[0 - \left(-\frac{1}{2K} \right) \right] = \frac{A^2}{K}.$$

So $A = \sqrt{K}$.

Therefore, confined to the potential, <u>there is only one bound energy eigenstate</u>:

$$\boxed{\varphi_E(x) = \frac{\sqrt{m\beta}}{\hbar} e^{-\frac{m\beta}{\hbar^2}|x|} \quad \text{with} \quad E = -\frac{m\beta^2}{2\hbar^2}}$$



E, ψ
E11
FORBIDDEN
Allowed
FORBIDDEN
E4
Allowed
FORBIDDEN
x
E, ψ
E10
FORBIDDEN
Allowed
FORBIDDEN
E5
Allowed
FORBIDDEN
FORBIDDEN
x

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5.33)

Notes:

- "Classically Forbidden" regions (in red) indicate anywhere where $E < V_0$, E is less than the potential.

- "Classically Allowed" regions (in blue) indicate anywhere where $E > V_0$, so E is greater than the potential.

- Where the potential is lower, the energy eigenstates φ_E will have more energy ($E - V_0$ will be greater if V_0 is smaller).

  This implies a higher kinetic energy ($K = E - U$), so $\frac{\partial^2}{\partial x^2}\varphi_E$ will be greater, implying a greater curvature in the wave.

  Likewise, the wave curves less when V_0 is larger ($\frac{\partial^2}{\partial x^2}\varphi_E \propto E - V_0$, $E - V_0$ is smaller $\Rightarrow$ smaller KE $\Rightarrow$ smaller / less curvature).

- Note the exponential decay of the wave when $V_0 > E$. This QM property is what violates classical particle behaviour.

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6.28) For $E > V_0$, the resonance transmission of a particle subject to the barrier is

$$E_n = V_0 + \frac{n^2 \pi^2 \hbar^2}{8ma^2}.$$

Here, $m = m_e = 9.11 \times 10^{-31}$ kg

$$a = \frac{1 \times 10^{-9}}{2} \text{ m}$$

$$n = 1, 2, \cdots$$

If $KE = E - V$, then $KE = V_0 - V_0 + \frac{n^2\pi^2\hbar^2}{8ma^2}$.

I will give the first 5 kinetic energies.

$$\left\{ \begin{array}{l} KE_1 = 6.0243 \times 10^{-20} \text{ J} \\ KE_2 = 2.4097 \times 10^{-14} \text{ J} \\ KE_3 = 5.4218 \times 10^{-14} \text{ J} \\ KE_4 = 9.6388 \times 10^{-14} \text{ J} \\ KE_5 = 1.5061 \times 10^{-14} \text{ J.} \\ \vdots \end{array} \right\}$$