The Photoelectric Effect

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Emre Alca 1005756193, Jace Alloway 1006940802

Abstract

The photoelectric effect is a well-known experiment due to its vital results: the discovery of the particle nature of light. JJ Thompson discovered that ultraviolet light, when fired at a metal target, produces an electric current. Many interpretations of this experiment were taken, such as Maxwell's theory of light and Albert Einstein's formulation, which lead to the conclusion of quantized light energy. This report compares these two explanations by examining the relationships between intensity and produced current (photocurrent), and determining whether there is a delay in the photocurrent after exposure to light. This work further consider the determination of Planck's constant, the 'work function' energy E_0 for ejection, and the frequency threshold of photocurrent cutoff f_0. The determined values were $h = (6.11 \pm 3.06) \times 10^{-34}\,\mathrm{J \cdot s}$, $E_0 = (2.08 \pm 0.18) \times 10^{-19}\,\mathrm{J}$, and $f_0 = 3.41 \times 10^{14} \pm 3.56 \times 10^{-6}\,\mathrm{Hz}$, with a curve fit percentage of 98 %. It was found that there is no significant delay between the light being emitted and a photocurrent being produced. The theoretical and experimental values determined for the photocurrent saturation time were $(7.95 \pm 0.68) \times 10^{-5}$ s and $(8.7 \pm 0.5) \times 10^{-5}$, respectively. It was lastly concluded that intensity and photocurrent are positively correlated, and hence all of the acquired results are as expected and reasonable.

Introduction and Theory

JJ Thompson's discovery of the photoelectric effect–that when ultraviolet light was fired at a metal target, the target would emit negative charges known as photoelectrons (hence the name)–and its competing explanations have played a vital role in demonstrating the discrete nature of light energy. An explanation for the effect was derived from Maxwell's theory of light: the energy in the electric field of a stream of UV light was continuously transferred to the metal target and its bound electrons. When an electron has absorbed more than a particular energy threshold (E_0), it would be launched out from the metal with some kinetic energy (E_k). This threshold E_0 was also called the work function and is characteristic to the specific metal being targeted. The kinetic energy E_k is also the difference between the total energy absorbed E_e and E_0:

$$E_k = E_e - E_0 \tag{1}$$

The explanation from Maxwell had the energy of light to be continuously distributed in a field. This field's intensity would be proportional to the square to the distance to the source, meaning in a weak field, or a field far away, would take some time to absorb an amount of energy equal to E_0, delaying the discharge of a photoelectron.

Philip Lenard studied this effect further. He used a two electrodes and a vaccum chamber to make an anode and cathode with a window to allow light to reach to cathode. Lenard measured the photcurrent (the current of photoelectrons) using an ammeter, as well the frequency of the light being emitted. By varying the voltage between the cathode and anode (ΔV), Lenard was able to measure E_k. From these measurements, Lenard was able to make four important observations:

(1) The intensity of the photocurrent is proportional to the intensity of the light hitting the target metal

(2) This current appears without delay, even if the light source is very dim

(3) Photoelectrons are only emitted if the frequency of the light is greater than some threshold: $f > f_0$ rather than the intensity of the light

(4) When ΔV is negative, the electrons are decelerated as they approad the anode. This decreases the photocurrent until it becomes zero (define this voltage to be $\Delta V = V_{\text{stop}}$ or the stopping potential).

The stopping pottential is independent of the intensity of the light source. This V_{stop} can be calculated as

$$V_{\text{stop}} = E_k/e \tag{2}$$

In 1905, Einstein published another paper providing another explanation to Lenard's observations. Einstein argues that energy does not 'flow' into the metal in a continuous fashion proportional to the intensity of the incident light. Rather, the energy arrives at the metal target in discrete packages with energy

$$E = hf \tag{3}$$

where $h = 6.63 \cdot 10^{-34}$ J · s (Planck's constant) and f is the frequency of the oscillation of the incident light. Here, if $E_0 \leq hf$, then a photocurrent can be measured and the kinetic energy of those photoelectrons is the energy surplus, as described by (1). Under the assumption that the intensity is not so high that a single electron to absorb more than one photon, (1), (2), (3) can be combined to say

$$eV_{stop} = hf - E_0 \iff V_{stop} = \frac{h}{e}(f - f_0) \tag{4}$$

This expression also provides a way to measure Planck's constant and f_0. We will calculate both of these quantities as well as the work function E_0.

Using equations (2) and (4), the time for the electron to be ejected into the anode can be determined by considering the power ratio which the electron absorbs. For a light of power P_{LED} incident on the cathode, the amount of power which is absorbed by the electron is

$$P_e = P_{\text{LED}} \left(\frac{A_e}{A_{\text{cathode}}} \right). \tag{5}$$

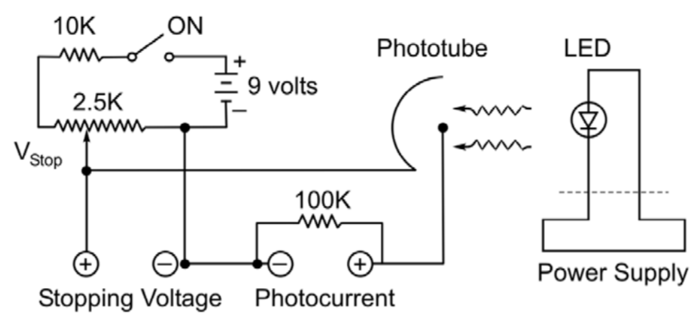
Since P_e has units of energy/time, then expression may be re-arranged by substituting E_0 in for E, therefore determining the amount of time that it takes for the electrons to be ejected. That is,

$$t = E_0 \cdot \left(P_{\text{LED}} \cdot \frac{A_e}{A_{\text{cathode}}} \right)^{-1}. \tag{6}$$

Since electrons are typically 3 nm apart, then the appropriate electron area is $\pi(3 \text{ nm})^2/4$, which is just the area of a circle which bounds the electron. There are two competing theoretical explanations for the photoelectric effect presented in this work: Maxwell's continuous field formulism, and Einstein's discrete energy explanation. This report will compare these formulations by analyzing whether or not there is a delay between light energy striking the metal cathode and the measurement of a photocurrent.

Methodology

This experiment made use of a power supply powering an LED, eight different LED's, a phototube, a 100 kΩ resistor (measured to be 98.9 ± 0.1 kΩ), a potentiometer, two multimeters, and an oscilloscope. The circuit was arranged as shown in Figures 1 and 2. The frequencies and spectral widths of the eight diodes can be found in Table 1, along with their respective uncertainties. The frequencies were previously measured and written on the diodes.



[Figure 1] The theoretical circuity used in this experiment. This circuit includes a power supply to an LED (providing the incident light, with 10 different diodes tested), a phototube (acting as the metal target), a 100 Ω resistor, a potentiometer, as well as two multimeters. A setup for a further experiment also requires an oscilloscope. The actual circuity is shown in Figure 2. This graphic is taken from [1].

The voltage reading of the photocurrent was measured as well as the stopping voltage using the two multimeters. To find the stopping voltage, the poteniometer was used to set the photocurrent to zero, giving a reliable measure of V_{stop}, since this is the voltage where electrons are no longer ejected. These values were saved in `.csv` files by hand. It is important to note that this current passes through a 100 kΩ resistor, converting the current into a voltage by Ohm's law. V_{stop} was measured for the 8 frequencies found in Table 1. The graphical results acquired of frequency vs V_{stop} can be found in figure 4.

Color	λ (nm)	$\Delta\lambda$ (nm)
Ultraviolet	390	40
Blue	455	40
Cyan	505	30
Green	535	30
Amber	590	10
Orange Red	615	10
Red	640	10
Infared	935	10

[Table 1] The frequencies and spectral widths of the eight diodes tested. Light from these diodes was fired at a metal target in a phototube to generate a photocurrent. These frequencies were previously measured and written on the diodes. In addition to these eight diodes, a variable intensity diode and an oscillator driven diode were used.

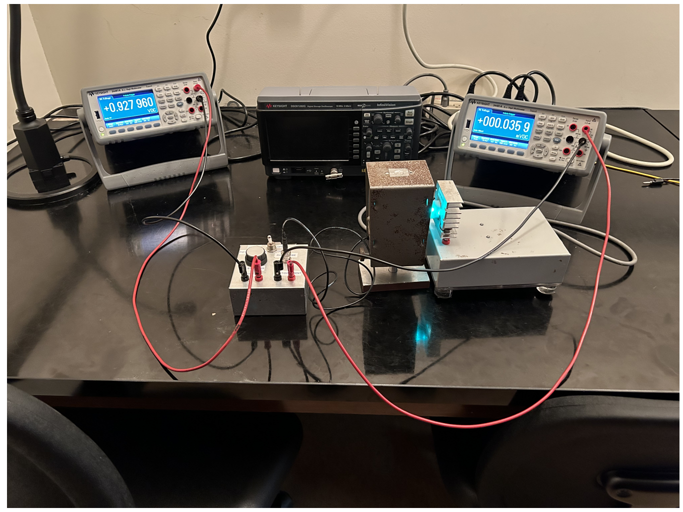
After measuring V_{stop} and frequency, a variable intensity LED was used. This variable intensity LED had 4 discrete steps, and these measurements were taken for each step. The change of intensity of the light itself was not measured.

A final experiment was performed examining the behaviour in the case of a LED being oscillatorily driven. The oscilloscope drives the LED amplitude into a square wave with the frequency 2.0 ± 0.1 kHz with 60 mW of power. The photocurrent and time were measured on the oscilliscope itself. These measurements will show whether there is a delay in the response of a photocurrent to changes in the intensity of incident light. Data from the oscilloscope was saved as `.csv` files onto a USB drive for analysis.

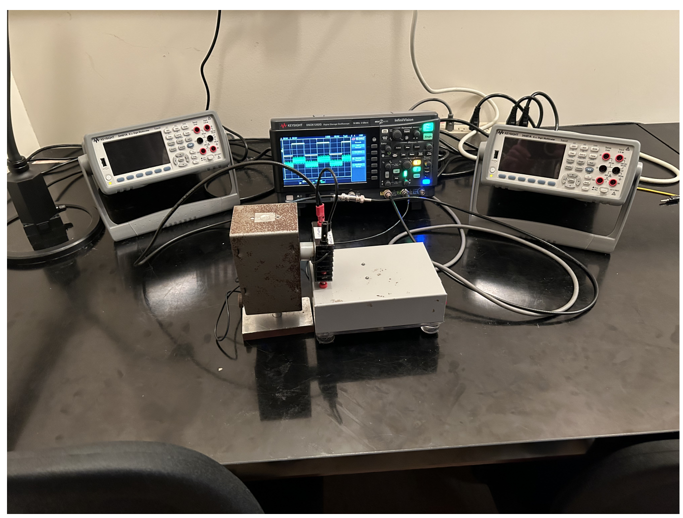
Data Analysis

The data analysis component was completed using Python. Throughout the duration of the data collection, the acquired data was tabulated in `.csv` files, which we then loaded into python using `pandas.load_csv`. This includes the data acquired from the oscilloscope.

For excercise one, the data was plotted on a V_{stop} - Frequency (Hz) graph, and then was fitted using a linear model function in the argument of `scipy.optimize.curve_fit` (see Figure FILL IN). This way, the values of V_{stop} and h (Planck's constant) could be easily extracted from the optimal fitting parameters. The uncertainties for h, in this exercise, were taken to be the maximum value of all the covariant fitting parameters, because the maximum covariant uncertainty to h corresponds to the error in the slope measurement automatically determined by curve fitting.



[Figure 2] The above aparratus was used to measure both photocurrent intensity and V_{stop} across different light frequencies and intensities. It has an LED powered by a power supply. This LED fires light into a phototube, generating a photocurrent. This photocurrent was passed through a potentiometer. The stopping voltage was measured as the potential difference across the potentiometer. The photocurrent passes through a 98.9 ± 0.1 kΩ resistor, and was measured by a multimeter in Volts. These measurements were recorded in .csv files for analysis.



[Figure 3] The above aprratus was used to measure the oscillatorily driven LED, and the delay in photocurrent response to changes to the intensity of incident light. An oscilloscope powers a diode by generating a square wave with the frequency 2.0 ± 0.1 kHz using 60 mW of power. The photocurrent was plugged back into the oscilloscope itself to be measured alongside the driving square wave. The data was saved from the oscilloscope as .csv files onto a USB

Equivalently, this was taken out for the optimal value of V_{stop}, since the maximum covariant uncertainty to this parameter depicts the maximum variation curve fitting could produce from the given data points. A simple χ^2 percentage fit was taken out to estimate the fit quality using scipy.stats.chi2.cdf(), where the uncertainties of the data points were used as a consistent control to measure overlap with the fit, instead of changing the fit by introduing the `pcov` variations for h and V_{stop}.

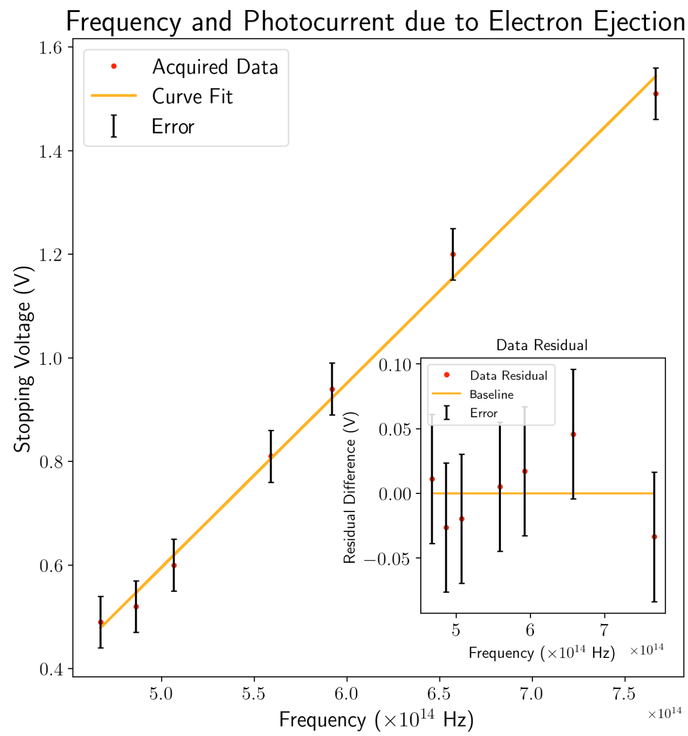
In excercise two, the intensity - photocurrent magnitude was plotted to measure the effects of light intensity on producing photocurrent. In general, no fit needed to be carried out since examining the trend of the fit was deemed sufficient enough. In general, the relationship of photocurrent versus voltage in terms of intensity could not be plotted due to a lack of data and the inability to collect such data from the limitations in the experimental design.

Lastly, all that was required in excercise three was to plot and interpret the data acquired from the oscilloscope. The data from both was plotted in a voltage-time graph, and the time it took for electrons to eject was manually determined by examining the graphs. To compare the observed value with the expected value, the determined V_{stop} from excercise one was taken and rearranged into equation (6) to determine time. The area of the electron was taken to approximately be $A_e = \frac{\pi}{4}(3 \times 10^{-7})\,\text{cm}^2$, and the area of the cathode was taken to be $3.23\,\text{cm}^2$.

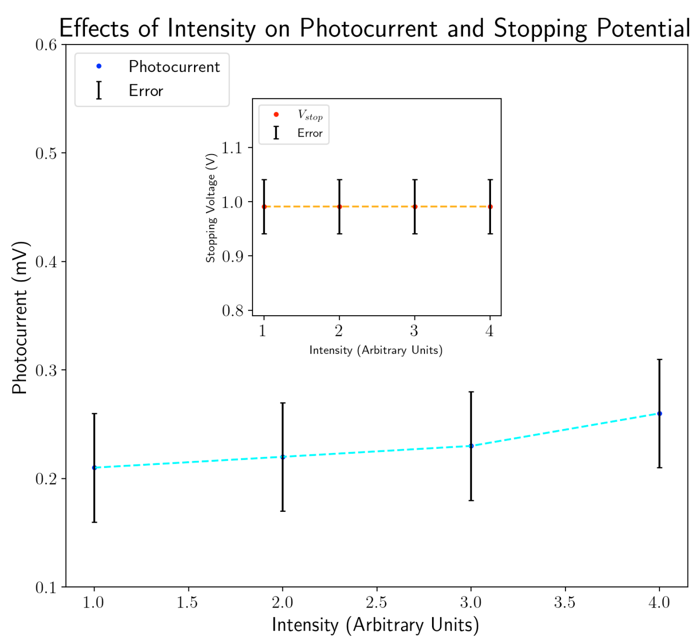
Results

The data corresponding to excercise one is plotted below in Figure 4.

From the curve fit, the value of Planck's constant was extracted along with it's uncertainty determined by `np.sqrt(np.diag(pcov))`. The y-intercept of the curve fit corresponded to the work function, since this is the value when the photocurrent vanishes according to equation (4). This value, along with its uncertainty, is recorded in Table 2. Division of the work function value by h yields the cutoff frequency, in Hz.



[Figure 4] Photocurrent due to electron current dependent on light frequency. The line, shown in orange, is the respective curve fit of the data. The frequency values of the light are on the order of 10^{14} Hz, hence the slope of the curve approaches Planck's constant when multiplied by electron charge e. The residual plot is depicted in the lower right, portraying the overlap of the errorbars with the appropriate fit.



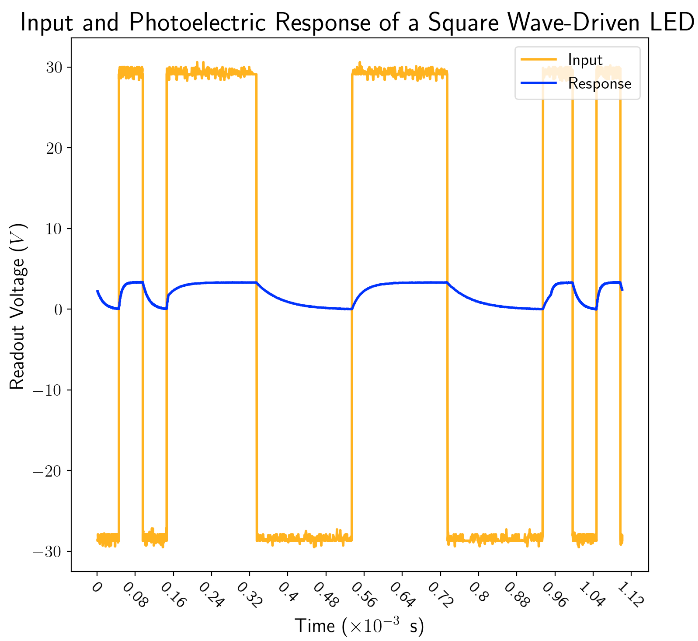
[Figure 5] The relationship between intensity of light and the generated photocurent. The inset plot indicates that the stopping potential is constant, while generally the photocurent increases as intensity increases. This is as expected.

Determined Constant	Value	Uncertainty
h	6.11×10^{-34} J $\cdot$ s	$\pm 3.06 \times 10^{-34}$ J $\cdot$ s
E_0	2.08×10^{-19} J	$\pm 1.79 \times 10^{-20}$ J
f_0	3.41×10^{14} Hz	$\pm 3.56 \times 10^{-6}$ Hz
χ^2	0.978 %	–

[Table 2] The measured curve fitting parameters respective to the linear fit shown in Figure 4. Planck's constant h was determined by measuring the slope of the line, while E_0 was the workfunction measured to be the y-intercept of the line. The value of f_0 was determined by dividing E_0 by h. The χ^2 value is depicted in the last column, shown as a percentage to depict the percentage quality of the linear fit.

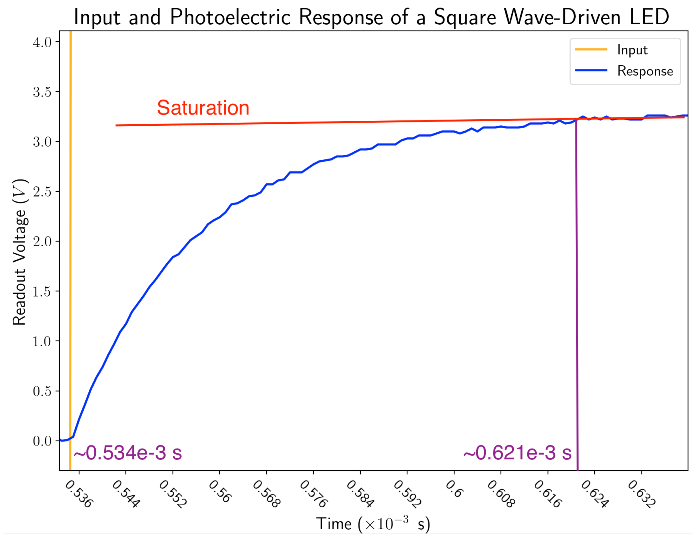
The results obtained by plotting the data acquired in excercise two is shown in figure 5.

Lastly, the results for excercise three are depicted in figure 6. Invoking relation (6), with $A_e = \frac{\pi}{4}(3\times10^{-7})^2$ cm^2 and $A_{\text{cathode}} = 3.23 \pm 0.01$ cm^2, and assuming that P_{LED} corresponds to all of the energy inputted into the light, which was taken to be 60.0 ± 0.1 mW, the time for the electron to be ejected was calculated.



[Figure 6] Input and response signals emitted and recieved by the oscilloscope. The input wave was a 2.0 ± 0.1 kHz square wave, while the response is due to the generated photocurrent in the phototube.

The time it takes for the electron to eject was calculated to be $7.95 \times 10^{-5} \pm 6.83 \times 10^{-6}$ s, which was compared with the observed value. This observed value was estimated by closely examining the plot shown in figure 6, such as zooming into the plot. This zoom is shown in figure 7.



[Figure 7] Input and response signals emitted and recieved by the oscilloscope, zoomed in. The input amplitude began at approximately $0.534\text{e} - 3$ s, and the photocurrent was saturated at around $0.621\text{e} - 3$ s.

The time difference for saturation is therefore approximately $8.7 \times 10^{-5} \pm 0.5 \times 10^{-5}$ s, where the uncertainty was taken to be reading error from the figure. It can be observed in figure 7 that there is no delay from the initial 'turning on' of the light to where electrons are ejected. This is as expected, considering Einstein's contribution to the problem.

Discussion

First, consider the neglegance of the infrared light in the excercise one results. It was observed that infrared light does not produce a significant photocurrent, since the frequency of infrared light is lower than the cutoff frequency necessary to eject an electron: $\frac{2.99 \times 10^8 \text{ m/s}}{935 \times 10^{-9} \text{ m}} = 3.19\times10^{14}$ Hz $< f_0 = 3.41\times10^{14}$ Hz (uncertainties have been neglected for brevity).

There are a few important sources of error in these results. The first comes from the fluctuations of the stopping voltage V_{stop} and photocurrent, generated by the voltmeters. Since these values are fluctuating significantly, we approximated their error to be ± 0.05 V, also accounting for the human error of dialing the potentiometer. Since there were only 7 light frequencies tested, the actual uncertainty in these values are likely greater than listed (though there is no confidence to exactly how much). Appropriate adjustments of these errors would therefore imply better measurements and produce more accurate results, such as reducing potentiometer error which would establish a more approximate value of h and E_0.

The results showed that Planck's constant is $(6.12 \pm 0.31) \times 10^{-34}$ J·s directly from `scipy.optimize.curve\_fit`. With uncertainty, this value does not agree with the value found in literature of $6.63 \cdot 10^{-34}$ [2], but instead is quite close to the expected value. This difference is most likely due to a lack of datapoints.

The work function was found to be $E_0 = (2.09 \pm 0.18) \cdot 10^{-19}$ J. This value does not change across any of our experiments, since it is an intrinsic property of the atoms in the metal cathode. The frequency threshold is also a function of the material, which, in this case, was found to be $f_0 = (3.14 \pm 0.01) \cdot 10^{14}$ Hz. This comes from equation (4), $E_0/h = f_0$. Overall, the fit generated in excercise one is good, which was concluded with a χ^2 probability of 97.8% of a good fit. The excercise one uncertainties are likely larger due to the lack of datapoints, as previously mentioned.

When examining Figure 5 in excercise two, there is no reasonable way to measure the change in the intensity of light directly, and only qualitatively observe the change in photocurrent. In the same figure, there is a positive nonlinear relationship between an increase in the intensity of light. One major change to this experiment would be to measure the different intensities of light directly to allow for a more quantitative analysis and hence characterization of the intensity-photocurrent relationship. Note that V_{stop} remains constant, since the frequency of the light does not change.

Excercise three measures the photoelectric response time by applying a square wave driven oscillatory LED to the cathode. The time it took to saturate the photocurrent is equivalent to the approximate amount of time it takes for electrons to be ejected from their nuclei by photon absorbtion. Assuming that every electron absorbs every photon, the area percentage relates to the fractional amount of energy which is absorbed by the atoms throughout time, which is power. Using formula (6), this time could manually be calculated as described in the previous section. Then, by examining Figure 7, an approximate measure of photocurrent saturation time could be determined. This value was compared with the data analysis value, and it was observed that the uncertainties obtained from each value overlap: $(7.95 \pm 0.68) \times 10^{-6}$ s, $(8.7 \pm 0.5) \times 10^{-5}$ s. It was hence concluded that these were reasonable values in estimating the electron ejection time in photocurrent saturation.

One limitation in this experiment is being able to measure oscillatory-driven LED's of various intensities, which may or may not reveal a relationship between the saturation time of photocurrent and the light intensity.

Considering the explanations formulated by Maxwell and Einstein, it can be concluded that Maxwell's reasoning is incomplete in examining the photoelectric effect. Maxwell's explanation assumes wave-like light, which fails to account for light-intensity and photocurrent saturation, since increasing the intensity of the light should, classically, produce a linear-time dependent photocurrent. This is not observed. Instead, photocurrent is saturated at a peak value, concluding that intensity produces a higher number of electrons instead of increasing their energies directly. Einstein, instead, suggests that light is particle-like instead of wave-like, which justifies the observations. This lead to the conclusion that light energy is discretely quantized into packets (called photons), and not continuous.

Conclusions

The results acquired in this experiment conclude the inadequacy of Maxwell's formulation of the photoelectric effect: that light is not wave-like, but instead discrete particle-like, as Einstein postulated. This conclusion is due to the observations made of the photocurrent-intensity dependence, and how photocurrent voltage peaks at a constant value instead of increasing in time, which is expected if light were wave-like.

The determined value of Planck's constant h in this experiment was approximately $(6.11 \pm 3.06) \times 10^{-34}$ J · s, which did not overlap with the expected literature value of 6.63×10^{-34} J · s. The work function and cutoff frequency, respectively, were found to be $(2.08 \pm 0.18) \times 10^{-19}$ J, $3.41 \times 10^{14} \pm 3.56 \times 10^{-6}$ Hz. The quality of such the curve fit was about 98 %, and the respective uncertainties were determined by human error in varying the potentiometer, and reading error due to the voltmeters. More error was attributed to a lack of data points and the limitations of the experimental setup. This concludes that the fit, along with their extracted values, were reasonable.

Overall, it was observed that intensity variation produced different, constant values of photocurrent voltage, which did not increase or change further in time. This proves that wave-like light theory is invalid, since an increase in light amplitude should continuously add energy into the photocurrennt, increasing it.

Lastly, the time delay for saturated photocurrent was measured using a square wave-driven oscillatory light emitting diode. It was found that the time for electrons to be ejected from the atoms to generate photocurrent was about $(7.95 \pm 0.68) \times 10^{-5}$ s theoretically and $(8.7 \pm 0.5) \times 10^{-5}$ s experimentally. These two values were consistent with one another, which justify a reasonable timescale. Photon-electron non-absorbtion was not accounted for, since it was assumed that every electron is ionized from the atom by absorbing light energy.

Bibliography

[1] *The Photoelectric Effect*. Department of Physics, University of Toronto.

[2] *Quantum mechanics*. McIntyre, D. H. (2022). Cambridge University Press.