Polarization of Light

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Abstract

This work investigates the properties of linearly polarized light. The goal of this experiment was to verify Malus's law for a series of two and three polarizers, and this was done using a laser diode, a detector, and rotatable acquisition polarizers exporting data to LabView. Overall, it was found that the output intensity for two polarizers was $I = 3.42 \pm 0.01$ W with a χ^2 fit value of $\chi^2 = 0.376$, and for three polarizers an output intensity of $I = 0.497 \pm 0.001$ W with a $\chi^2 = 0.426$. The second component of this experiment was to determine the index of refraction of an acryllic D-lens, which was done by determining Brewster's angle from observing reflected and transmitted polarized light. It was found that the observed index of refraction was $n_{\text{acryllic}} = 1.5 \pm 0.5$, which was compared to a value found in literature, of $n = 1.49$. The reflection and transmission coefficients were determined by the Fresnel equations, which were observed to be $R = r_\perp^2 = 0.140 \pm 0.56$ and $T = r_\parallel^2 = 0.865 \pm 0.221$. In conclusion, the values which were observed were reasonable and acceptable, which verifies Malus's law and the index of refraction of acryllic.

Introduction

In introductory optics and electromagnetism, light polarization is the instrinsic property of electromagnetic waves which is given by the orientation of propogation. Generally, there are three types of light polarization: linear, circular, and elliptical. Most light is linearly polarized, and may be manually polarized by means of a polarizer, by reflection, scattering, or refraction through denser media.

Maxwell's equations predict the linear polarization of light as perpendicular electric and magnetic field components, which then are both perpendicular to the direction of the propogation. This allows various projections of these components on vertical and horizontal axes, hence polarizing the light. Any light passing through a polarizer is then polarized in the direction of the polarizer.

Today, polarizers are used in sunglasses, laser physics, photography, and other ranges of electromagnetic waves (radio, x-rays, gamma rays, etc).

The goal of the following experiments was to verify Malus's law for linearly polarized light, and to determine the index of refraction for an acryllic D-lens by measuring Brewster's angle.

The necessary preparation for this experiment was reading on the physics of linearly polarized light, Maxwell's equations, Snell's law, and Brewester's angle.

Theory

In the absence of electric charge and current distributions, Maxell's equations may be rearranged and re-substituted to obtain the wave equations for typical electric and magnetic field components:

$$\Box^2\, \mathbf{E}(\mathbf{r}, t) = 0, \qquad \Box^2\, \mathbf{B}(\mathbf{r}, t) = 0, \tag{1}$$

where $\Box^2 \equiv \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is the D'Alembertian operator. Solving these equations by D'Alembert's method provides expressions for $\mathbf{E}$ and $\mathbf{B}$ as

$$\mathbf{E}(\mathbf{r}, t) = \text{Re}\left\{\boldsymbol{\mathcal{E}}_0 \exp\left(i\frac{\omega}{c}(\mathbf{n}\cdot\mathbf{r} - ct)\right)\right\} \tag{2.1}$$

$$\mathbf{B}(\mathbf{r}, t) = \text{Re}\left\{\boldsymbol{\mathcal{B}}_0 \exp\left(i\frac{\omega}{c}(\mathbf{n}\cdot\mathbf{r} - ct)\right)\right\}, \tag{2.2}$$

where $\boldsymbol{\mathcal{E}}_0$ and $\boldsymbol{\mathcal{B}}_0$ are the electric and magnetic wave directions, respectively, ω is the angular frequency of the wave, and $\mathbf{n}$ the direction of propogation.

Then, (2.1) and (2.2) are related by Faraday's Law,

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{3}$$

which yields the electro-magnetic wave relation

$$\text{Re}\left\{\left(i\frac{\omega}{c}\mathbf{n} \times \boldsymbol{\mathcal{E}}_0 - i\omega\boldsymbol{\mathcal{B}}_0 \exp\left(i\frac{\omega}{c}(\mathbf{n}\cdot\mathbf{r} - ct)\right)\right)\right\} = 0, \tag{4}$$

which is true for all space and time components. Therefore $\boldsymbol{\mathcal{B}}_0 = \frac{1}{c}\mathbf{n} \times \boldsymbol{\mathcal{E}}_0$, resulting in perpendicular electric and magnetic field components of a light wave. Polarization is the effect of projecting these vector components of (4) onto a plane, reducing the light intensity and specifying a polarized direction.

This report focuses on two interesting effects of light polarization: Malus's law, and reflectance polarization in the form of Brewster's angle. Malus's law models the effects of polarizers aligned in series to each other, and how the intensity of the light changes.

In its simplist form, Malus's law states that the intensity of light as it passes through two polarizers is proportional to $\cos^2\theta$, where θ is the angle between the two polarizers. Suppose the polarizer is oriented in the $\hat{y}$ direction, and let the transmission axis of this second polarizer be $\hat{y}'$. In this case, the components of the wave which pass through this second polarizer is

$$E_{x'} = E\sin\theta \qquad\qquad E_{y'} = E\cos\theta \tag{5}$$

with the $\hat{y}'$ component being transmitted only (since $\hat{x}'$ is orthogonal to the transmission axis of the second polarizer). By definition, since intensity is proportional to the square of the electric field component, $I_0 \propto E^2$ (the intensity between the polarizers), thus the intensity of light transmitted by both of them is

$$I(\theta) = E^2\cos^2\theta = I_0\cos^2\theta \tag{6}$$

This expression is called Malus's Law. For this experiment, θ is known from our measurements, and $I(\theta)$ will be extrapolated using an optimization algorithm.

For two polarizers perpendicular to each other, then, it is expected and observed that the output intensity is indeed zero:

$$\begin{aligned} I(\pi/2) &= I_0\cos^2(\pi/2) \\ &= 0. \end{aligned} \tag{7}$$

If a third polarizer is placed further along the $\hat{z}$ axis such that its transmission axis is orthogonal to that of the first polarizer, some intensity does, interestingly enough, transmit through. The intensity that passes through this third polarizer can be found by applying Malus's law a second time in series. If the intensity of light passing through the first polarizer is I_1, then the intensity through the analyzer is

$$I_1 = I_0\cos^2\varphi \tag{8}$$

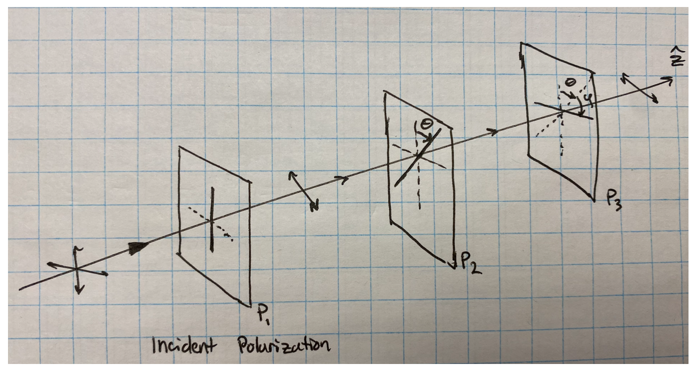
where φ is the angle between the transmission axes of the polarizer and analyzer. Then $\theta + \varphi = \frac{\pi}{2}$. Applying Malus's law a second time,

$$I_2 = I_1\cos^2\left(\frac{\pi}{2} - \varphi\right) \tag{9.1}$$

$$= I_0\cos^2(\varphi)\cos^2\left(\frac{\pi}{2} - \varphi\right) \tag{9.2}$$

$$= \frac{I_0}{4}\sin^2(2\varphi) \tag{9.3}$$

A representation of this phenomenon is shown in Figure 1.



[Figure 1] The effect of three-polarizer polarization on an incident beam of vertically-polarized light.

In the same way, light can be polarized by means of reflection and transmission upon incidence on different types of media. For reflections on air-glass interfaces, the Fresnel equations provide expressions for the parallel and perpendicular reflectance coefficients $r_\parallel$ and $r_\perp$:

$$r_\perp = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} \tag{10.1}$$

$$r_\parallel = \frac{n_1\cos\theta_2 - n_2\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1} \tag{10.2}$$

$$r_\parallel^2 + r_\perp^2 = 1. \tag{10.3}$$

In practice, these coefficients relate the reflected and transmission angles (θ_1 and θ_2, respectively) with the refraction indices (n_1 and n_2), which are also related by Snell's law of refraction

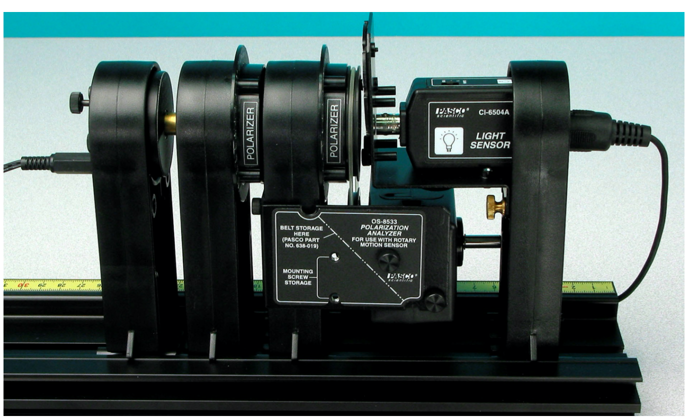
$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}. \tag{11}$$

The angle at which the reflectance coefficient $r_\perp^2$ is zero is called Brewster's angle, and is given by $\tan\theta_p = \frac{n_2}{n_1}$. This yields a value of n_2, and by equation (11), the transmission angle θ_2 can be determined, thus determining the transmission coefficient $r_{\parallel}^2$, which is the amount of light transmissed upon polarization with the medium.

Experimentally, for vertically polarized light, θ_p can be determined by reading off from data from where the reflected horizontally polarized intensity in terms of θ_1 is zero, and this determines Brewster's angle and hence the index of refraction of the medium n_2. These processes are described in the methodology.

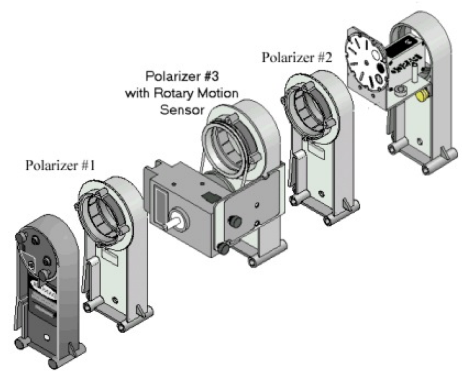
Methodology

The following experimental trials were performed in a dark room with minimal light. To begin, a small 10-volt laser beam was attached to the end of a long track with two polarizers in series with one another. At the end of the track, a laser-acquisition device was placed with a collomating slit in front of it to filter some of the light out to avoid over-saturating the detector. The detector gain level was set to '1', and one of the polarizers was linked to LabView. It was manually rotated 2π radians while LabView acquired the data. These measurements were repeated until the best, smoothest curve was measured. The apparatus is shown in Figure 2.



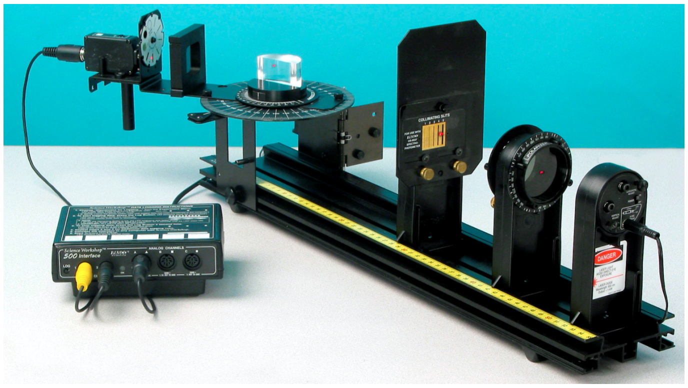
[Figure 2] Experimental setup and components of the two-polarizer apparatus. The apparatus consists of a laser diode, two polarizers, one of which is exporting data to LabView, and a diode detector.

Once a sufficient amount of data was collected for two polarizers, a third polarizer was added behind the acquisition lens, which was then removed to calibrate the second polarizer. This was done by adjusting the second polarizer so that its direction was orthogonal to that of the first polarizer, and a minimal amount of light was being detected in LabView. The acquisition polarizer was then re-added, and LabView was set to record 2π radians as the center polarizer was manually adjusted. Again, this experiment was repeated until the observed data was consistent, uniform, and smooth. The apparatus is depicted in Figure 3:



[Figure 3] Depiction of the experimental setup used in verifying Malus's law for three polarizers. The apparatus consists of three polarizers, one of which exports angle data to LabView, a laser diode, and detector.

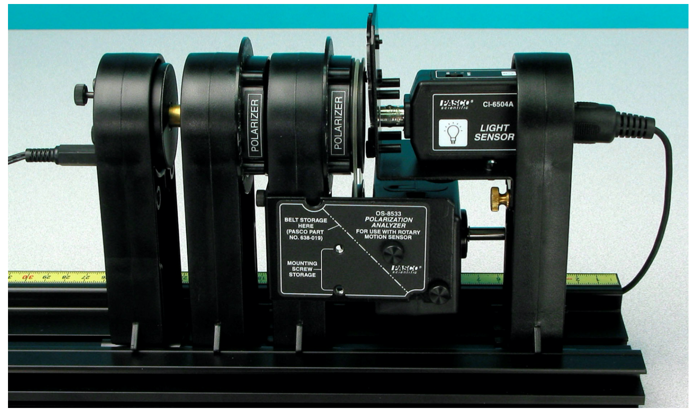
The last set of trials which were performed were to determine Brewster's angle. The laser detector gain was configured to '10' and a collomating slit was placed in front to avoid over saturating the detector. The detector was attached to a long, rotatable arm. In the center of the rotator was an acryllic D-lens, which angle information was read by an acquisition device and exported to LabView. Incident to the D-lens was diode-laser light which had been polarized and run through a collomating slit to control the amount of light let through. The initial polarizer was the control, and was left in the horizontally-polarized position which was determined from the trials on Malus's law. The experimental apparatus is shown in Figure 4.



[Figure 4] Experimental setup for determining Brewester's angle. The apparatus consists of a rotatable D-lens, a laser diode, a polarizer, collomating slits and a detector.

To determine Brewester's angle, the laser diode was turned on and one individual rotated the D-lens (whose data was exporting into LabView) while the other person rotated the laser detector on the rotatable arm. A second trial was performed, where a horizontal square polarizer was placed on the end of the rotating arm in front of the detector, and a third trial was performed with a vertical polarizer. Overall, these trials took many attempts to minimize the random error in the data which was acquired. All three of these trials were completed effectively, however only the horizontal polarizer data would be needed to complete the Brewster's angle analysis.

These three trials were performed individually, as the first trial was used as a control data set to observe the effects of reflective and transmittive polarization on observed light intensity, as the first trial was perform polarizer free. Since the incident light to the D-lens was horizontally polarized, measuring the reflected horizontally polarized light with the square polarizer would then determine Brewster's angle: the value of θ for which the intensity $I_{\perp}$ was measured to be zero. In the third trial, the square polarizer was rotated to be vertical, and the previous method was again executed to compare the values of $I_{\|}$ with the expected values, determined from the initial trial of $I_{\perp}$. These intensities are related by the Fresnel equation (10.3).



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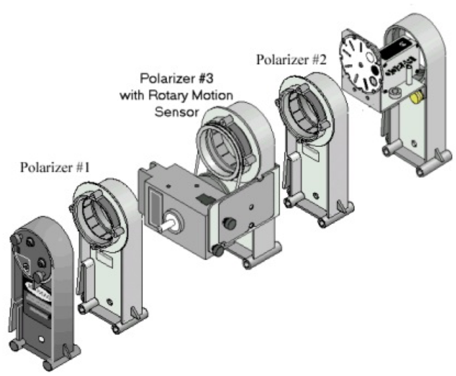

[Figure 3] Depiction of the experimental setup used in verifying Malus's law for three polarizers. The apparatus consists of three polarizers, one of which exports angle data to LabView, a laser diode, and detector.

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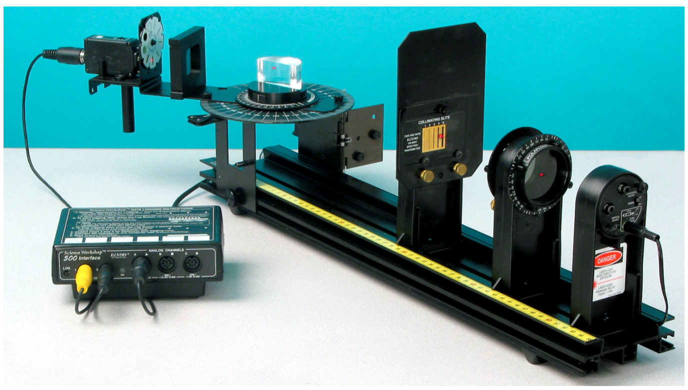

[Figure 4] Experimental setup for determining Brewester's angle. The apparatus consists of a rotatable D-lens, a laser diode, a polarizer, collomating slits and a detector.

Data Analysis

Malus's Law will be discussed first. The data, as recorded in *Labview*, was stored in separate .txt files for each of the two trials (2 and 3 polarizers). These files were read by the python library *numpy*'s *.readtxt*() function, and isolated by column for *Intensity* (*V*) and *position* (*radians*). The raw data was plotted using another python library, *matplotlib*, with uncertainties (see the upper charts of Figure 5 and Figure 6 for the plots of the 2- and 3-polariod trials respectively). The uncertainties for each of these charts was one half of the smallest significant digit. The plot for 2-polarids was fit using an equation in the form of (6), and for 3-polaroids was fit using an equation in the form of (7) using *scipy.optimize.curve_fit*(). Each of these functions had additional parameters for a phase-shift and an intensity offset.

The function fit the 3-polaroid trial quite easily, but the 2-polaroid trial was a little bit more tricky. The flatter tails at either end of the sinusoidal wave were culled from the dataset, and the function was able to fit (see Figure 5 to see the comparison between the original and culled data).

The results of these fits are discussed in the *Results and Discussion section below*. The uncertainties of the relevant parameters of these fits (for these functions, I_0 and I_1 for the 2- and 3-polaroid trials respectively) were found by taking the square root of the covariance of the optimized parameters found by *scipy.optimize.curve_fit*. So long as the maximum value of the curve generated by these optimized parameters (as well as the majority of the curve itself) fit within the uncertainties of the measured data, the parameters are sufficient, within the uncertainty of the measurements. The residual plots of each trial (the difference between the fit curve and the measured data) were also plotted (the residual plots can be found in the lower charts of Figure 5 and Figure 6).

Results and Discussion

For Malus's law, the model found $I_0 = 3.42 \pm 0.01$ V and a phase shift of -0.985 ± 0.003 radians with a $\chi^2 = 0.376$ for the 2-polaroid trial. For the 3-polaroid trial the model found $I_1 = 0.497 \pm 6.8 \cdot 10^{-4}$ V and a phase shift of $1.16 \pm 7.3 \cdot 10^{-4}$ with a $\chi^2 = 0.426$. It is important to notice that, with three polarizers, the intensity is at its maximum when $\varphi = \pi/4$ and at a minimum when $\varphi = 0, \pi$. This might be hard to notice from the graph, but when considering the phase shifts in mind, these values align well with the peaks and troughs of the graphs in Figure 6. Another important and interesting thing about these results, is that while we managed to find a good fit, there is some strange behavious in the plot of Intensity vs. $\cos^2\theta$ (figure 7). This graph is expected to be linear, but actually has this strange wing-like shape (see Figure 7). This is surprising since Intensity should be linearly related to $\cos^2\theta$, but is not.

For Brewster's angle, it was found that $\theta_p = 92.3 \pm 0.3°$, which makes $n_2 = 1.5 \pm 0.5$. With these values, it was calculated that $r_\perp = 0.37 \pm 0.28$, which makes $r_\parallel = 0.93 \pm 0.11$. The sum of the squares of these two refraction coefficients is 1.0 ± 0.0, which is as expected. All of $n_2, r_\perp, r_\parallel$ are dimensionless. The result found in literature for n_2 is 1.5 [2], which is well within the uncertainty found here. An interesting thing to notice about the relationship between n_2 and θ_p is that $\theta_p = \arctan(n_2/n_1)$. This means that the larger n_2 is, the smaller θ_p is. From [4], we know that

$n_{\text{water}} = 1.33$, and thus is less than n_2. This means that Brewster's angle at the interface between water and air would be less than the θ_p found here. We also explored the the results of the apparatus with a vertical polarizer (i.e. both of the polaroids are aligned). The results of this can be found in Figure 8, showing a much slower and smoother decay of intensity. If this was with a set of polarizers, the shape would be sinusoidal, but the intensity does not return after reaching 0 V due to the same effects that keep the intensity at 0 V for the two opposed polaroids after reaching Brewsters angle. This discission of decay has an interesting effect on polarized sunglasses. Polarized sunglasses are optimized to filter light at a particular angle–say, light reflecting off of water at about 2 meters (low angles). These sunglasses will block glare coming off the water very well, but if the same sunglasses are worn by a pilot looking at the same reflection from the sky, she would likely get little benefit since the angle is well above Brewster's.

Conclusions

Overall, Malus's law was verified in all cases, despite some strangeness in the 2-polarizer setup. Reasonable values for the refraction index of acryllic, and its resulting refraction coefficients were found, and found to be coherent.

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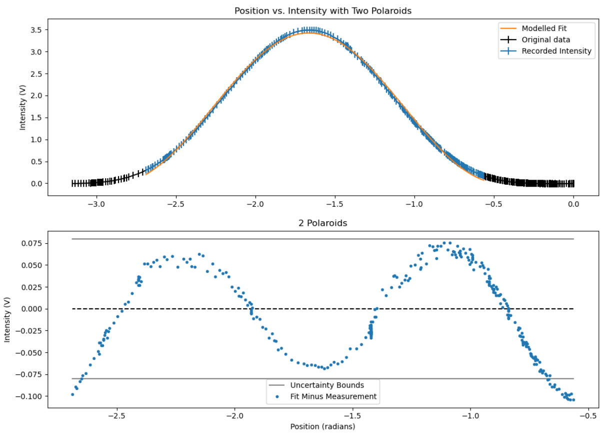
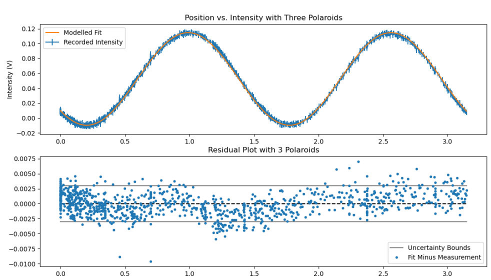
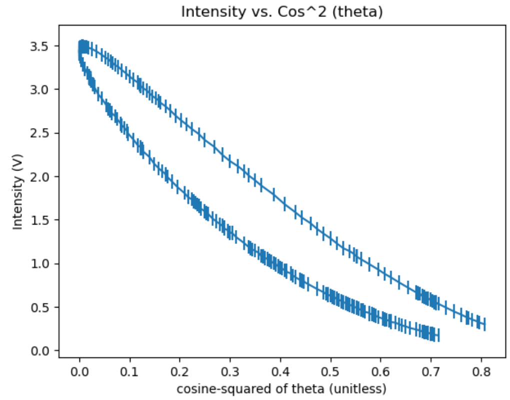
Appendix I: Figures and Tables



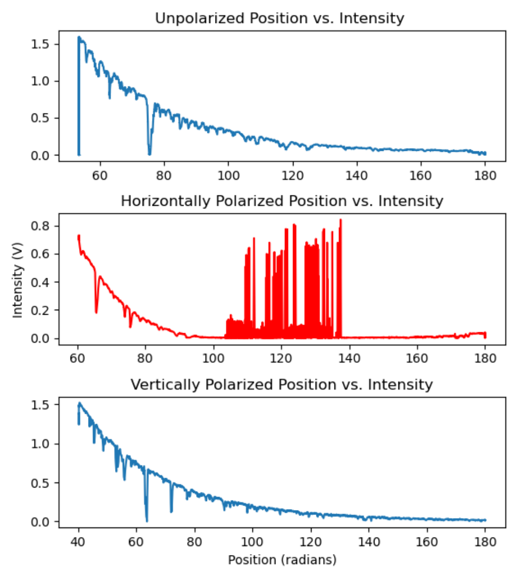
[Figure 5] Observed data for the verification of Malus's law for two polarizers. The above plot, on the vertical axis, the output intensity, and the angle in radians on the horiztonal axis. The $\chi^2 = 0.376 < 1$, which makes this an excellent fit for the data. The lower figure displays the residual data plot with uncertainty overlap.



[Figure 6] Observed data for the verification of Malus's law for three polarizers. The above plot, on the vertical axis, the output intensity, and the angle in radians on the horiztonal axis. The $\chi^2 = 0.497 < 1$, which makes this an excellent fit for the data. The lower figure displays the residual data plot with uncertainty overlap.



[Figure 7] The graph of Intensity vs. $\cos^2(\theta)$ should have a linear slope. Instead, as seen here, it has this strange nonlinear behaviour.



[Figure 8] The data collected in the 3 trials for finding Brewster's angle are found above. The upper plot is the decay of intensity when there is no box polarizer in front of the detector. The second graph is when the two polarizers are orthoginal relative to one another, and in the third, bottom, graph the two polarizers are parallel. Notice that in all three cases the intensity decays as more of the light is refracted rather than reflected, and the intensity does not return. There are some significant sources of uncertainty present in these graphs, but none of them have a significant effect on our analysis.