# Two-Terminal Devices

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Emre Alca 1005756193, Jace Alloway 1006940802

**Abstract**

The purpose of this experiment is to use Ohm's Law and Shockley's equation to fit the voltage vs. current relations of 10 different devices across 11 trials. For many of these devices (two resistors, a battery, a short circuit, a thermistor at room temperature and body temperature), Ohm's law fits the relation well. However, for others (four kinds of diode and a selenium rectifier), Ohm's law fails quite spectacularly (with $\chi^2$ values as high as 6.14)–very far from the acceptable range of 0 to 1. Shcokley's equation, however, is able to fit the voltage vs. current relations for all of these devices.

## Introduction

In circuitry and electrical engineering, resistance models are often used to quantify the relationships between input current and voltage differences in a circuit. Circuits typically consist of a power source, resistors and diodes, and loads such as light bulbs. The goal of this experiment was to attempt to characterize and compare various voltage models to observed phenomena in a simple circuit. These models were taken from physical laws which describe how current and voltages are related in resistors and diodes, such as Ohm's law and the Shockley diode equation.

## Theory

In resistor loads, the current and voltage relationship may be linearly modelled by Ohm's Law:

$$V = IR, \tag{1}$$

where $V$ is the voltage difference across the resistor, $I$ is the current input and $R$ is the resistance. In many loads, this linear relationship holds, however it was observed in diodes that the voltage-current relationship is in fact exponentially non-linear. A diode is a two-terminal device which only conducts electricity in one direction. A diode itself consists of an anode and a cathode, and therefore acquires a high resistance in the opposite direction of current flow. Named after a transistor developer, the Shockley diode equation accurately depicts resistance behaviour in diodes as

$$I_D = I_S \left( \exp\left[ \frac{V_D}{V_T\, n} \right] - 1 \right), \tag{2}$$

where $I_S$ is the input current, $V_D$ is the voltage across the diode, $V_T$ is the thermal voltage (voltage distributed by heat production due to resistance), and $n$ is the ideality factor, which is 1 in this experiment and therefore may be omitted. The alternating current junction applied on the ciruit in this experiment allows for accurate current and voltage observations from various diodes and resistance loads.

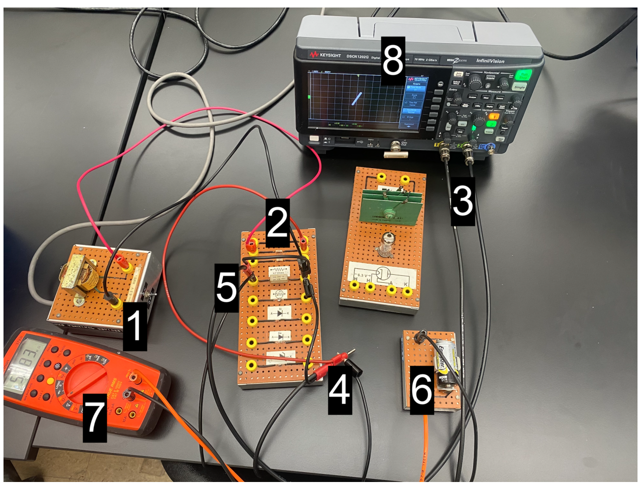
## Methodology

To determine the voltage differences across resistors and diodes by applying a current, first a 60 Hz $6.30 \pm 0.10$ V transformer was plugged in to produce an alternating current. The output was then run through a $99.1 \pm 1\,\Omega$ resistor using banana cables as a control so that current measurements for all loads would be ideally identical. Then, the voltage was measured across the selected device (resistor or diode) by using coaxial cables into the $x$ and $y$ CRO inputs of a Keysight oscilloscope. The circuit was then closed through a $4.70 \pm 0.01\,\Omega$ resistor into the oscilloscope ground and back into the transformer using banana cables. A depiction of the experimental setup is shown in Figure 2, while photos of the setup are shown in Figure 1.

The data for each trial was exported onto a USB through the oscilloscope into a '.csv' file. The data acquired contained timestamp measurements for both the input X and Y voltages as well as their respective measurements. Devices were changed by unplugging the banana cables from the previous device and plugging them back into the new device, and the data acquisition process was completed for each of the 11 various resistors, loads, and diodes. Invidiual datasets were compiled according to each load trial.

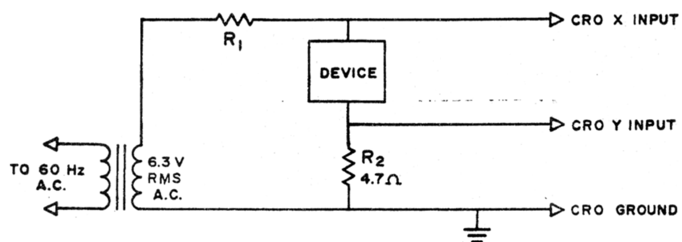
| Device | Measured Resistance ($\Omega$) | Calculated Resistance ($\Omega$) | Scale Current $I_S$ (A) | Thermal Voltage $V_T$ (V) | Ohm's Law $\chi^2$ | Shockley Eq. $\chi^2$ |
|---|---|---|---|---|---|---|
| $46.8 \pm 0.1\Omega$ Resistor | $46.8 \pm 0.1\Omega$ | $11.2 \pm 0.1\ \Omega$ | $30 \pm 10$ A | $300 \pm 200$ V | 0.16 | 0.16 |
| $993 \pm 1\Omega$ Resistor | $993 \pm 1\ \Omega$ | $194 \pm 9\ \Omega$ | $0 \pm 10$ A | $(1 \pm 2) \cdot 10^3$ V | 0.11 | 0.11 |
| Battery | $(5.08 \pm 0.01) \cdot 10^6$ | $1.98 \pm 0.01\ \Omega$ | $(0 \pm 1) \cdot 10^5$ A | $(0 \pm 2) \cdot 10^5$ V | 0.16 | 0.16 |
| Thermistor (Room Temperature) | $(1.17 \pm 0.01) \cdot 10^3\ \Omega$ | $156 \pm 7\ \Omega$ | $0.9 \pm 0.6$ A | $(1.4 \pm 0.9) \cdot 10^2$ V | 0.13 | 0.13 |
| Thermistor (Body Temperature) | $(1.17 \pm 0.01) \cdot 10^3\ \Omega$ | $143 \pm 5\ \Omega$ | $0 \pm 9 \cdot 10^4$ A | $(0 \pm 1) \cdot 10^6$ V | 0.11 | 0.11 |
| Short Circuit | $0 \pm 1\ \Omega$ | $1.00 \pm 0.01\ \Omega$ | $0 \pm 10 \cdot 10^4$ A | $(0 \pm 10) \cdot 10^4$ V | 0.40 | 0.40 |
| Silicon Diode | $(3.18 \pm 0.01) \cdot 10^6\ \Omega$ | $31 \pm 3\ \Omega$ | $(1.3 \pm 0.1) \cdot 10^{-2}$ A | $(3.83 \pm 0.04) \cdot 10^{-1}$ V | 6.14 | 0.54 |
| Germanium Diode | $(1.65 \pm 0.01) \cdot 10^6\ \Omega$ | $47 \pm 3\ \Omega$ | $(3.9 \pm 0.9) \cdot 10^{-2}$ A | $1.94 \pm 0.02$ V | 2.22 | 0.27 |
| Xenon Diode | $(2.71 \pm 0.01) \cdot 10^6\ \Omega$ | $5.9 \pm 0.7\ \Omega$ | $(1 \pm 1) \cdot 10^{-2}$ A | $(3.75 \pm 0.06) \cdot 10^{-1}$ V | 6.10 | 0.67 |
| Vacuum Diode | $(4.61 \pm 0.01) \cdot 10^6\ \Omega$ | $60 \pm 3\ \Omega$ | $(5 \pm 1) \cdot 10^{-2}$ A | $3.15 \pm 0.03$ V | 0.97 | 0.14 |
| Selenium Rectifier | $(18.0 \pm 0.1) \cdot 10^3$ | $39 \pm 3\ \Omega$ | $(5 \pm 1) \cdot 10^-2$ A | $1.69 \pm 0.01$ V | 2.81 | 0.23 |

[Table 1] Results obtained for each device: of measured resistance, calculated resistance (from Ohm's Law), Scale current and Thermal Voltage ($I_S$ and $T_V$ from Shockley's equation), and the $\chi^2$ values for Ohm's Law and Shockley's equation. Notice from the $\chi^2$ values that there are some devices for which Ohm's law is a satisfactory model (the two resistors, the battery, the thermistor, and the short circuit). Technically, Ohm's law is just barely a satisfactory model for the vacuum diode as well ($\chi^2 = 0.97$), but, from its plot in Figure 4, its relation is clearly nonlinear. For the other devices (the four diodes and the selenium rectifier), Ohm's law is clearly a bad model, with $\chi^2$ going as high as 6.14. In these cases, Shockley's equation is a satisfactory model. In the cases where Ohm's law is a good model, the uncertainties of the parameters of Shockley's equation are very large even though its $\chi^2$ value effectively equal to that of Ohm's law. This is likely because the exponential factors are very small causing the possible error to explode. The measured and calculated resistances also do not agree, likely due to some unmeasured scaling factor.




[Figure 1] The circuity constructed in this experiment. (1) The 6.3 Volt AC transformer. (2) The input resistor, at $100\,\Omega$. (3) The oscilloscope CRO X and Y inputs. (4) The device terminals, currently configured to a short-circuit (no resistance). (5) The output $4.7\,\Omega$ resistor. (6) A battery load, whose resistance is being measured by (7) the ohmmeter. (8) The Keysight oscilloscope and display, currently showing the current-voltage relationship for the short circuit in XY display mode.

10 devices were tested in 11 trials: a $46.8 \pm 0.1\Omega$ resistor, a $993 \pm 1\Omega$ resistor, a battery, a thermistor at both room and body temperature, a short circuit, a silicon diode, a germanium diode, a xenon diode, and a slenium rectifier.




[Figure 2] The theoretical circuity used in this experiment. This circuit includes an alternating current producing transformer, two resistors, a diode device, and oscilloscope inputs. The actual circuity is shown in Figure 2. Image is taken from [1]

### Data Analysis

To analyze the data acquired from the oscilloscope, it was first loaded into a fresh python file using `pandas.read_csv()`. Each trial was labelled accordingly, and a dictionary was used to configure between the linear resistance load relationships and the exponential Shockley relationships. This was done using a simple 'for' loop. Using `scipy.optimize.curve_fit`, each trial was fitted according to Ohm's law (Eq. (1)) and the Shockley diode equation (Eq. (2)). Residuals were calculated and the uncertainties in current observations for each of the trials were taken to be the baseline width of each data set (i.e. their inherent noise), since the width of each data set line acquired from the oscilloscope yields a variation in data acquisition and experimental error from the scope. For each fit, a $\chi^2$ residual was computed to determine the quality of each fit.

### Results

The data measured by the oscilloscope for each device can be found in Figure 3 (for the two resistors, the battery, the two thermistor trials, and the battery) or in Figure 4 (for the four diodes and the selenium rectifier). These figures also contain the fit curves derived from both Ohm's law and Shockley's equation. The explicitly measured resitances, as well as the calculated factors and $\chi^2$ values of each curve can be found in Table 1. The major results of this report will come from comparing the $\chi^2$ values of Ohm's law and the Shockley equation for the various devices. The chi squared value quantifies how well a model fits data. If a fit is good, 2/3 of the datapoints in the residual will be within uncertainty of 0. When this is true, $0 < \chi^2 < 1$. If $\chi^2 \leq 0$, then the model is complex than the data, and the model is most likely overfitting the data and the important relationship is missed. When $1 \leq \chi^2$, the model is underfit and the important relationship is still being missed (often, when this is true, the fit curve does not even follow trend of the data visually). According to the $\chi^2$ values, Ohm's law was a valid model for every device included in Figure 3, as well as the vacuum diode (just barely). For the devices in Figure 4, the $\chi^2$ value of the curves drawn by Ohm's law are not acceptable. In all cases, however, Shockley's equation is a satisfactory model. An interesting thing to note about the results of Shockley's equation is that the uncertianties of the scale current $I_S$ is seemingly absurdly large for the cases where Ohm's law holds. It is also important to note that even in cases of good fits, the measured resistances do not agree with the values calculated from Ohm's law.

### Discussion

As described in the results section, the important relationships of this report lie in the $\chi^2$ values of Ohm's law and the Shcokley equation for the devices tested. The devices represented in Figure 3 are all modelled properly by both Ohm's law and Shockley's equation, with all of their $\chi^2$ values falling in the acceptable range of $0 < \chi^2 < 1$. In each of these cases, the chi squared values are actually identical and the two models draw what is effectively the same line. This is because the relation between voltage and current, in these cases, is nicely linear. An interesting thing about these cases is that the uncertainties for $I_S$ in Shockley's equation is absurdly large. The size of these uncertainties make good sense from the fact that these relations are linear. This means that $V_D$ must be effectively zero, making large changes in $V_T$ inconsequential.

For the devices in Figure 4, there is a clearly visible exponetial relationship between current and voltage in the data. In these cases, the curves fit by Ohm's law are very bad models of the data. This is represented in the laughably poor $\chi^2$ values (going as high as 6.14), and how visibly poorly a linear curve fits these results. Shockley's equation fits them very nicely though–and these plots exhibit nicely switch-like behaviour, as Shockley would have needed for the devleopment of transistors.

There is also the exceptional case of the vacuum diode, where Ohm's law has an acceptable $\chi^2$ value of 0.97 despite it's plot showing a nonlinear relationship. The acceptability of this fit likely comes from the fact that this device is the most weakly nonlinear of those tested, with it having the shallowest slope after diverging from 0 A of current.

The uncertianties of the results shown in these figures may seem surprisingly large. The uncertainty of $\pm 0.05$ V was chosen rather than the uncertainty of the oscilloscope itself due to noise inherent to the electronic system. This uncertainty is quite consistently visible, especially in the results shown in Figure 3, and their presence persists to Figure 4.

There is one especially surprising result in this report: that the calculated values of the resistance of these devices universally fails to agree with their measured counterparts. This is likely because the procedure outlined in this report fails to consider the scaling of the oscilloscope, which likely has some effect on the results. Thus, in each case there should be come scaling factor that should be applied to the calculated resistances to normalize them, and bring them into agreement with the measurements.

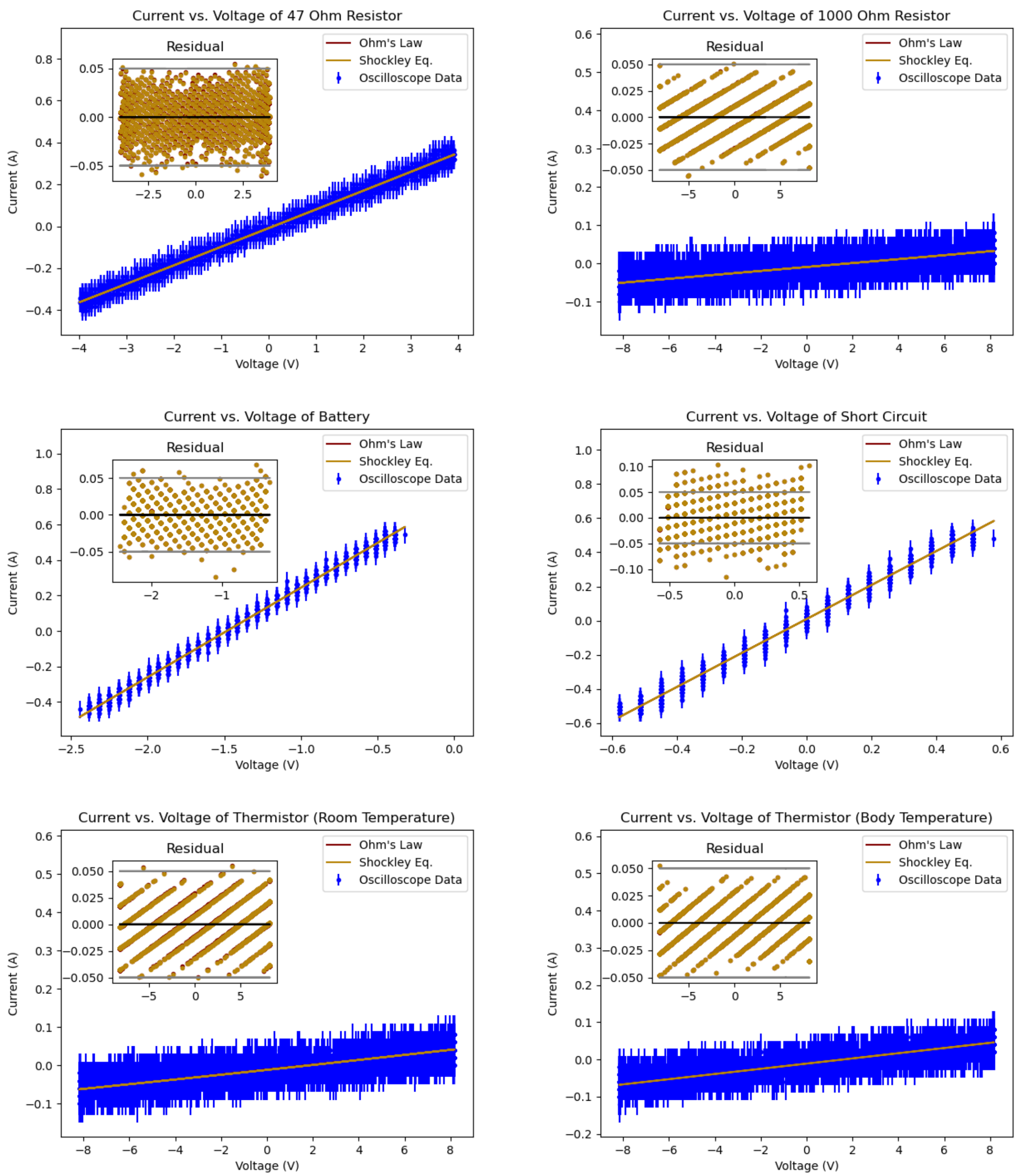
### Conclusions

In this experiment, the performance of Ohm's law and the Shockley equation were directly compared across 10 devices in 11 trials. For six of these trials (a $46.8 \pm 0.1\Omega$ resistor and a $993 \pm 1\Omega$ resistor, a battery, a short circuit, and a thermistor at both body and room temperature) Ohm's law performed well. The intensity of the current was linearly related to the voltage, as expected. However, for the other five trials (a Silicon diode, a Germanium diode, a Xenon diode, a vacuum diode, and a Selenium rectifier), the relationship between current and voltage is nonlinear. In the nonlinear cases, Ohm's law fails as a model. The data is fundamentally more complex than the model. Shockley's equation works much better in these cases. This report shows that the Shockley equation models the cases where Ohm's law fails as well as the cases where Ohm's law holds.

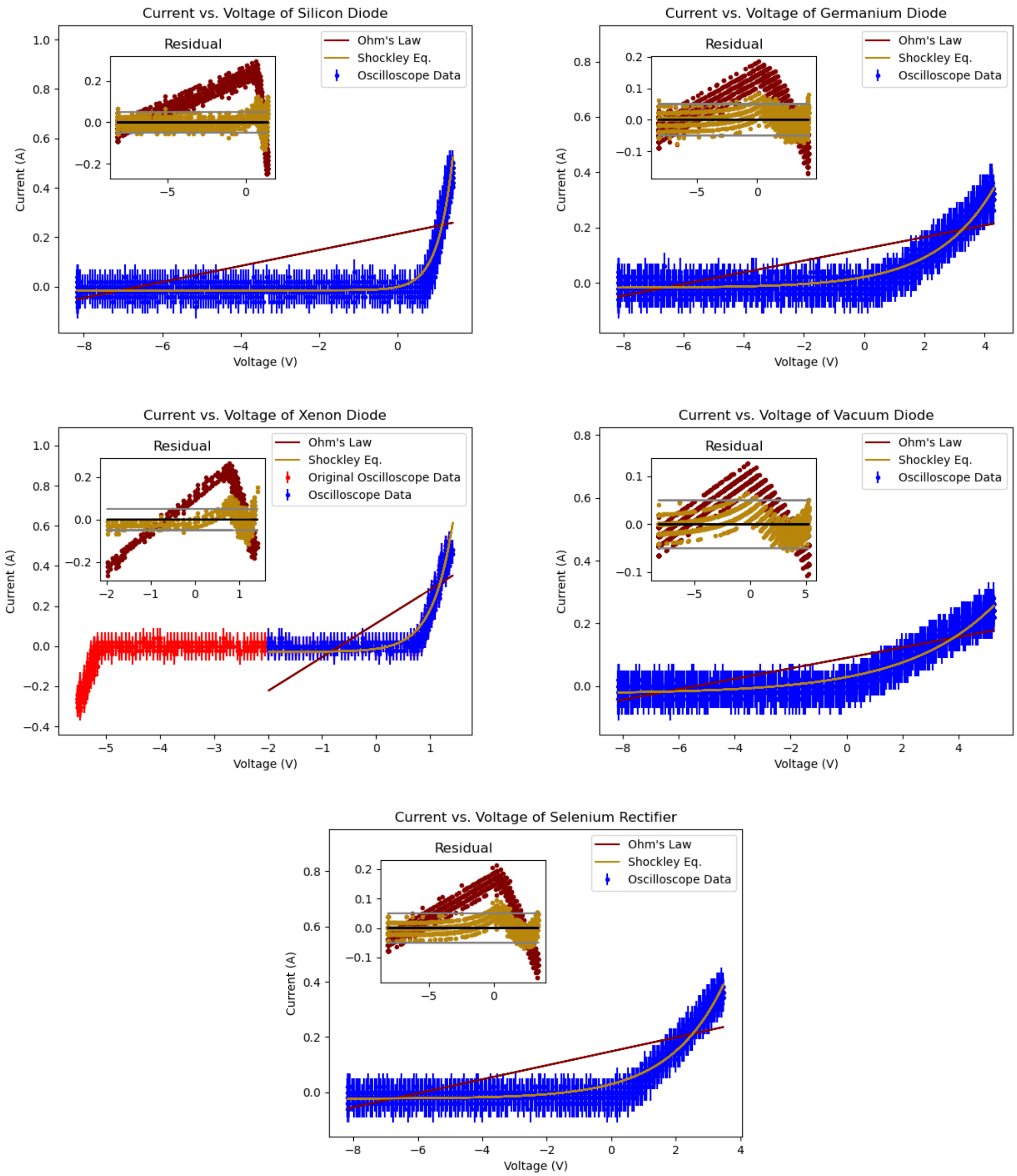
## Bibliography

[1] *Two-Terminal Devices - Principally Diodes*. Department of Physics, University of Toronto.

## Appendix I: Figures and Tables




[Figure 3] The above plots show the results for the devices with linear relationships between current and voltage (i.e. those for which Ohm's law is a good model). These are also cases where the Shcokley equation is a good mdoel, since an exponential function can be made linear when its exponant goes to zero. In fact, the two models draw almost the exact same line and share $\chi^2$ values, as seen in Table 1. It is important to note that the residual plots share the axis with their parent plots, being current vs. voltage. The uncertainties of the data for these plots is due to noise ineherent to the electronic system.




[Figure 4] The above plots show the results for the devices with nonlinear relationships between current and voltage (i.e. those for which Ohm's law fails as a model). These plots exhibit some very clearly exponential behaviour, making the linear fits to the data almost comical. The $chi^2$ values of these plots can be as high as 6.14 (as seen in Table 1), which is very bad. However, Shockley's equation fits very nicely. The difference in effectiveness at fitting the data is also very clear in the residual plots, where the results of Ohm's law are outside of the reasonable error (grey lines) much more than those of Shockley's equation. It is important to note that the residual plots share the axis with their parent plots, being current vs. voltage. The uncertainties of the data for these plots is due to noise ineherent to the electronic system. For the xenon diode, only half of the data was fit, since its two sides are the same, only reversed in sign. Therefore, to fit the data, only the positive half was considered. The original data in in red, and the used data is in blue like the rest of the plots.