MAT235 Tutorial 13

TA: Jace Alloway (j.alloway@mail.utoronto.ca)
Office Hours: T13-14 BA2145

Tutorial this week will focus on parametrizing curves and evaluating line integrals. Aside from the problems (posted below), there are some brief announcements and Quiz 10 will take place.

1. Let C be the curve consisting of an arc C_1 defined by the function $y = x^2 - x$ for $x \in [0, 1]$ and the line segment C_2 between the points $(1, 0)$ and $(0, 0)$.
   (a) Find parametrizations for the curves C_1 and C_2.
   (b) Let $\mathbf{F}(x, y) = \langle -y, x \rangle$ be a vector field. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ using the parametrizations found in part (a) (the 'o' over the integral sign represents a line integral over a closed loop).
2. Let $\mathbf{F} = (x + 2y)\mathbf{i} + (x - y)\mathbf{j}$. Let C_1 be the line segment from $(0, 0)$ to $(2, 2)$ and let C_2 be the curve parametrized by the function $\mathbf{r} : [0, 2] \to \mathbb{R}^2$ given by $\mathbf{r}(t) = \langle t, 0.5t^2 \rangle$ (explicitly, $0 \leq t \leq 2$).
   (a) Find $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ and $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$.
   (b) Compare the values of the line integrals of $\mathbf{F}$ over C_1 and C_2. What does this suggest about $\mathbf{F}$?
   (c) Repeat the above calculations for the vector field $\mathbf{G}(x, y) = (x + 2y)\mathbf{i} + (2x - y)\mathbf{j}$. What changed?