MAT235 Tutorial 10

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This week: Term test 3 review. Refer 16.4 - 16.5 Hughes-Hallett.

Summary of coordinate transformations and differentials: https://en.wikipedia.org/wiki/Del_in_cylindrical_and_spherical_coordinates.

Problems (taken up in tutorial):

1. Use polar coordinates to find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

2. Write the following sum of triple integrals as one iterated triple integral. Then, evaluate the integral.

$$\int_0^{\pi/2} \int_0^1 \int_{\sqrt{1-r^2}}^{\sqrt{4-r^2}} r^2 \cos\theta \, dz dr d\theta + \int_0^{\pi/2} \int_1^2 \int_0^{\sqrt{4-r^2}} r^2 \cos\theta \, dz dr d\theta.$$

3. Let E be the solid region above the xy-plane which is contained between two concentric spheres: one of radius a and the other of radius b, where $0 < a < b$. Use spherical coordinates to determine a formula for vol(E) in terms of only a and b (no integrals).

4. Use cylindrical or spherical coordinates to find the volume of the solid W which lies between the paraboloid $z = 24 - x^2 - y^2$ and the cone $z = 2\sqrt{x^2 + y^2}$.

Additional Problems (solutions posted by Wednesday next week):

5. Show that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$. *This is called the Gaussian integral, I. Hint: extend the integral into two-dimensions, then transform into polar coordinates. We can write*

$$I^2 = \int_{-\infty}^{\infty} e^{-x^2}\,dx \int_{-\infty}^{\infty} e^{-y^2}\,dy.$$

6. The plane $x - y + z = 2$ intersects the cylinder $x^2 + y^2 = 4$ in an ellipse. Find the points on the ellipse closest to and furthest from the origin. *Hint: to optimize distance $\sqrt{x^2+y^2+z^2}$ in $\mathbb{R}^3$, we can optimize $f(x,y,z) = x^2 + y^2 + z^2$ as it's derivative is simpler.*

7. Let $H = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4,\ z \geq 0\}$ be the upper solid hemisphere of radius 2. Express the volume of H as an iterated double integral with integration order $dydx$.

8. Consider the solid S above $z = 0$, below $z = y + 2$, and below the paraboloid $z = 4 - x^2 - y^2$. Use slices to construct the volume of S in terms of iterated integrals with the order $dxdydz$. Explain the geometry of your process. Do not evaluate the integral.

9. Consider the parallelogram $P = \{(x,y) \in \mathbb{R}^2 : 0 \leq x - y \leq 3, 1 \leq x + y \leq 3\}$ and its mass-density function $\delta : P \to \mathbb{R}$ given by $\delta(x,y) = (x+y)^2$. Use a change of variables to find the mass of P.

10. Evaluate $\iint_R (2x - \pi y)\,dA$ where R is the parallelogram with vertices $(2,0), (5,3), (6,7), (3,4)$. *Hint: derive a change of variables.*

11. Find the extreme values of the function $g(x,y) = x^2 + y^2 - 2x + 3y + 5$ on the region $D = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4\}$.

12. Let T be the part of the solid cylinder $x^2 + y^2 \leq 6$ which lies above the xy-plane and below the sphere $x^2 + y^2 + z^2 = 16$. Evaluate the triple integral

$$\iiint_T f(x,y,z)\,dV$$

where $f(x,y,z) = x + z$.