MAT235 Tutorial 4

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This week: Term Test 1 (Friday, October 18) review. Refer 12.1 - 12.6, 13.1 - 13.4, and 14.1 from Hughes-Hallett.

Problems (taken up in tutorial):

1. Consider three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$. Which of the following quantities are scalars, vectors, or undefined?

   (a) $\mathbf{v} \cdot \mathbf{w}$

   (b) $\mathbf{v} \times \mathbf{w}$

   (c) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

   (d) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

   (e) $\dfrac{\mathbf{v} \times \mathbf{w}}{\mathbf{v} \cdot \mathbf{w}}$

   (f) $\dfrac{\mathbf{v} \cdot \mathbf{w}}{\mathbf{v} \times \mathbf{w}}$

2. Consider three points $P = (-2, 0, 4), Q = (-1, 1, 3), R = (0, 3, 5)$.

   (a) Find a unit vector $\mathbf{u}$ which is orthogonal to the plane passing through P, Q and R.

   (b) Verify that $\mathbf{u}$ is orthogonal to the vectors $\mathbf{PQ}$ and $\mathbf{PR}$.

3. Consider a point $P = (0, 1, 2)$ and the line L parametrized by $\mathbf{r}(t) = \langle 2, 2, -1\rangle + t\langle 0, -2, 1\rangle$. Find the equation of a plane in standard form which contains both P and L.

4. Which of the following statements are true or false? If the statement is true, provide a brief justification or proof. If the statement is false, provide a counterexample. Consider three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$.

   (a) If $|\mathbf{v}| > |\mathbf{w}|$, then $|\mathbf{v} \times \mathbf{u}| > |\mathbf{w} \times \mathbf{u}|$.

   (b) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$ for all $\mathbf{u}$.

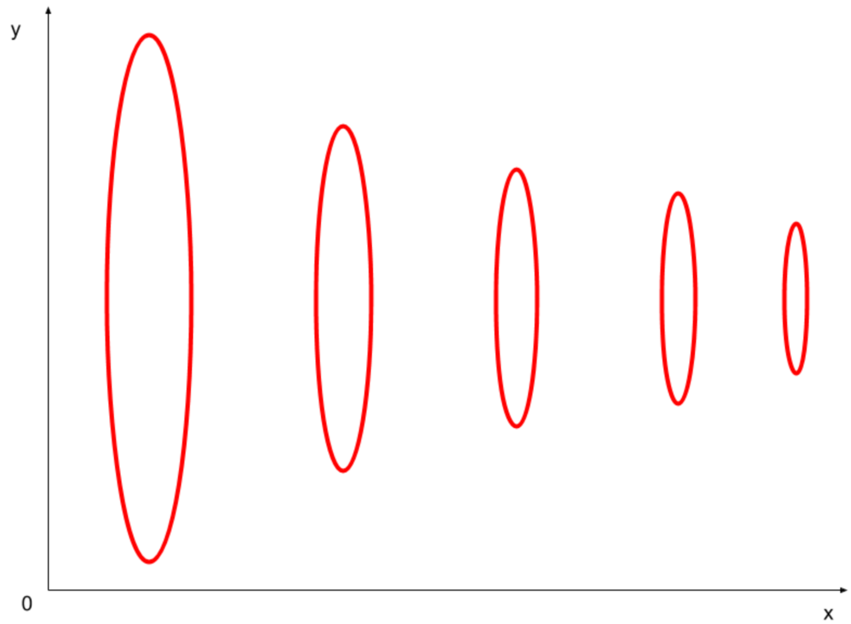
   (c) $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$ for $\mathbf{u} \neq \mathbf{v}$.

   (d) $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

   (e) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.

Additional Problems (solutions will be posted by Wednesday next week):

5. Attempt to write a function whose contours at $z = 0$ are similar to



   Hint: I would suggest doing this in Desmos with an implicit function. There is no exact answer. You may place conditions on the domain of the function. The goal of this problem is to identify the relationships between variables, transformation operations, and implicit functions with contours.

6. Let P be a plane containing the points $(4, -3, 1), (-3, -1, 1)$ and $(4, -2, 8)$. Let L be a line parametrized by
$$\mathbf{r}(t) = \langle 10, 9, 5 \rangle + t\langle 5, 3, 1 \rangle.$$
   Determine whether P and L intersect. If they do, find the point of intersection.

7. Evaluate the limit
$$\lim_{(x,y)\to(0,0)} \frac{x^2 y^3}{x^5 + y^3}.$$

8. Evaluate the limit
$$\lim_{(x,y)\to(0,0)} \frac{y^3 e^{-y^2} \sin(xy)}{xy}.$$

9. Consider the heat distribution along a single axis $U(x, t)$ (such as a wire). The distribution of heat over time is goverened by the heat equation,
$$\frac{\partial U}{\partial t} = \kappa \frac{\partial^2 U}{\partial^2 x},$$
   where κ is the thermal diffusivity constant. You can solve for $U(x, t)$ given a set list of initial conditions (called "boundary conditions") on $U(x, t)$. Explain the meaning of the derivatives U_t and U_{xx} and how their spatial and temporal components are related.

For example, a relatively simple solution to the heat equation, for $0 \leq x \leq L$ and $t \geq 0$, is

$$U(x,t) = 6\sin\left(\frac{\pi x}{L}\right) e^{-\kappa(\pi/L)^2 t}$$

where the explicit boundary conditions are $U(x,0) = 6\sin\left(\frac{\pi x}{L}\right)$, $U(0,t) = U(L,t) = 0$.